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HYBRID STATE ESTIMATION APPROACH TO MULTIOBJECT TRACKING FOR AIRBORNE SURVEILLANCE RADARS

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HYBRID STATE ESTIMATION APPROACH TO MULTIOBJECT TRACKING FOR AIRBORNE SURVEILLANCE RADARS

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October 1984

Submitted to:

Department of the Navy
Naval Air System Command
Washington, D.C. 20360

Contract Number N00019-82-C-0456

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: TR-180	2. GOVT ACCESSION NO.
	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle): Hybrid State Estimation Approach to Multiobject Tracking for Airborne Surveillance Radars	5. TYPE OF REPORT & PERIOD COVERED
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): R. B. Washburn, T. Kurien, A. L. Blitz, A. S. Willsky	8. CONTRACT OR GRANT NUMBER(s): N00019-82-C-0456
9. PERFORMING ORGANIZATION NAME AND ADDRESS: ALPHATECH, Inc. Burlington, MA 01803	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS: Naval Air Systems Command Washington, DC 20361	12. REPORT DATE: October, 1984
	13. NUMBER OF PAGES: 99
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report): Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Multiobject Tracking
Hybrid State Estimation
Airborne Surveillance

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report describes the application of hybrid state estimation theory to develop a unified theory of airborne surveillance. Hybrid state estimation provides a framework for treating estimation problems in which both continuous, numerical-valued variables and discrete, logical-valued variables occur. This framework allows one to model many different surveillance problems (such as multiple maneuvering targets, formation flying, missile launch and interception) and to construct the corresponding tracking algroithm. This report examines such models and investigates the resulting optimal and suboptimal tracking algorithms.

ABSTRACT

This report describes the application of hybrid state estimation theory to develop a unified theory of airborne surveillance. Hybrid state estimation provides a framework for treating estimation problems in which both continuous, numerical-valued variables and discrete, logical-valued variables occur. This framework allows one to model many different surveillance problems (such as multiple maneuvering targets, formation flying, missile launch and interception) and to construct the corresponding tracking algorithm. This report examines such models and investigates the resulting optimal and suboptimal tracking algorithms.

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OVERVIEW

1.1 INTRODUCTION

Keeping track of the positions and identities of multiple objects is an essential part of naval airborne command and control. In response to the general need for such capabilities in air surveillance, many different multitarget tracking algorithms have been proposed over the past two decades. Some of these are based on well-established principles of statistical estimation theory. Such an algorithm usually addresses a precisely defined mathematical problem having a precisely defined mathematical solution, although feasible implementation forces the algorithm only to approximate the theoretically optimal solution. Many more algorithms, especially those currently in actual operation, are based on ad hoc methods, that is, on experience and intuition applied to specific features of the particular operational situation, but without the essential guidance of a precisely formulated statistical theory.

Theoretically-based methods clearly point out that optimal algorithms are not feasible in realistic environments (with <u>any</u> imaginable computational hardware), and that a large number of suboptimal approximations are possible. It is also clear that there is a great variety of operational situations (types of targets, types of sensors, signal environment, etc.), and that there must be a corresponding variety of multitarget tracking algorithms suitably tuned to their applications. Thus, the requirement of feasible implementation

and the need for operational specificity both encourage the development of many different types of multitarget tracking algorithms. There is presently a significant need to categorize these different types of algorithms and to unify the methods used to develop them. Our research on tracking algorithms for airborne surveillance is motivated by this need. Our overall objective is to develop a unified theory of multitarget tracking that indicates which suboptimal approximations sacrifice the least performance for a given model of the operating environment.

Our approach to developing such a unified theory has been to formulate the multitarget tracking problem as a hybrid state estimation problem (i.e., a problem with both continuous and discrete states to estimate), to implement the optimal solution of the hybrid state estimation problem, and to study methodically approximations of the optimal algorithm and the resulting performance loss. We have applied this approach to study problems with a single position sensor (approximating a single active radar) with measurement errors, an unknown number of maneuvering targets which can appear and disappear at unknown times, missed detections, and false detections.

We have also applied this approach to study more complex surveillance problems such as the detection and tracking of aircraft flying in formation, aircraft launching missiles, and missile interceptions. The approach and results are described in detail in this report. In this section we give an overview of the report and summarize our work.

1.2 HYBRID STATE MODEL

We have found it useful to view multitarget tracking in terms of a stochastic dynamic system whose state consists of a continuous part x and a

discrete part z. The generic hybrid state model we use has the form (state and observation equations)

$$x(t+1) = f(t,x(t),z(t),w(t))$$
$$y(t) = h(t,x(t),z(t),v(t))$$

where $w(t),v(t)$ are process and measurement noises. We have assumed that z is a discrete (denumerable) state Markov chain that is not influenced by the continuous state x. To date we have used mainly the linear Gaussian hybrid state model in which f,h are linear functions of x,w, and v. Note that in multitarget tracking models it is convenient to allow the dimensions of x(t) and y(t) to change in time and the number of discrete states to be infinite in order to model new targets appearing at random times.

In our formulation the continuous state consists of positions and velocities for each existing target, and the discrete state consists of target dynamic modes (for maneuvers), target status (birth or death), false and missed detection indicators, number of returns at time t, and association of targets with returns at time t. The measurements consist of noisy position observations and number of returns per scan.

The hybrid state model provides a very general and convenient representation of stochastic dynamic systems. This approach allows us to represent continuous, numerical-valued variables and discrete, logical-valued variables in a systematic, unified framework. We feel that this ability to treat continuous and discrete variables together in a systematic way makes the hybrid state approach a powerful technique for modeling surveillance problems, and ultimately, for designing surveillance systems.

1.3 TRACK-ORIENTED ALGORITHM

The optimal hybrid state estimation algorithm computes the posterior probability of each discrete hypothesis and the corresponding conditional expectation of the continuous state variables. The mathematical structure of the optimal algorithm is well-known, as are its severe computational requirements. Despite the exponentially (or faster) growing number of computations, we nevertheless found it enlightening to construct a computer implementation both as a starting point for designing practical suboptimal algorithms and as a performance benchmark to compare algorithms in carefully selected test situations.

The optimal algorithm was constructed using the concept of hypothesis trees. In this approach every possible combination of measurements with existing tracks, new targets and false alarms is created at every stage. Each such combination is referred to as a global hypothesis. It is constructed sequentially based on the global hypotheses in the previous scan and the measurements in the current scan. There are several ways in which the set of new global hypotheses can be created. Techniques that have been used in the past are based on the creation of a global hypothesis tree which grows with each scan. The terminal branches (leaves) of the tree represent all the global hypotheses at the end of the current scan. The expansion of the global hypothesis tree is based on either a target-oriented approach or a measurement-oriented approach. Neither method can represent the growth of the hypothesis tree in a clear manner to reflect all aspects (track initiation, track termination, false alarms, missed detections, etc.) of multiobject tracking. The approach that we have adopted, the track-oriented approach described in this report, is unique in that we do not contruct a global hypothesis tree.

Rather, we maintain a list of global hypotheses and a set of target trees. The root of each target tree represents the birth of the target and the branches represent the different associations of this target with measurements available in subsequent scans. A trace of successive branches from a leaf to the root of the tree corresponds to a potential track of the target. The leaf of each such trace is unique and it is referred to as a track node of the target tree. Each element of the global hypothesis list contains a set of pointers which point to track nodes. In essence, they represent the combination of target tracks postulated in that global hypothesis.

Suboptimal techniques for managing hypotheses in the general hybrid state estimation algorithm are straightforward extensions of gating, pruning, merging, and clustering. These fit naturally into the track-oriented structure. However, the quantitative value of these different suboptimal techniques is still unclear. We have performed numerical simulations which give some qualitative picture of the effectiveness of a technique, but much remains to be done in determining quantitative effectiveness.

The track-oriented data structure described above is a very efficient structure for representing the type of hybrid state systems arising in multi-object tracking problems. This structure is convenient both for new hypothesis generation and for likelihood and filter computations. Furthermore, although we have not studied this aspect in depth, the track-oriented data structure provides a natural parallel computational structure for array processing.

1.4 MANEUVERING TARGETS

We show in this report that the hybrid state multiobject tracking algorithm can easily incorporate the tracking of multiple, maneuvering targets.

The hybrid state framework is flexible enough to formulate the problem of multiple targets which are "born" (i.e., appear) and "die" (i.e., disappear) at unknown times, and which can maneuver between straight and accelerated motion during the interval in which they are "alive." The model also includes the possibility of missed detections of a present target, false detections of clutter, and the unknown association of returns in any one scan with targets in track. We also consider the effect of nonlinear radar measurements (i.e., range and azimuth) on the tracking of multiple maneuvering targets.

The additional hypotheses generated to account for the possibility of maneuvers makes it impractical to run the optimal algorithm for more than two or three scans. Thus, we use gating, pruning, and a simple form of merging to reduce the number of hypotheses generated. Two approximations are particularly useful to deal with multiple maneuvering targets. The first approximation is to prevent initiation of maneuvers until the target's velocity error is sufficiently small. This screening technique prevents the inclusion of tracks which postulate maneuvers as a consequence of the large gate sizes at the time of track initiation. The other useful approximation is to prevent hypothesizing several different maneuvers in succession. Note that both of these techniques for practical implementation can be thought of as resulting from specific hybrid state modeling assumptions. For example, the first results from assumptions about the number of detections one expects before the target can transition to a maneuver state, and the second approximation corresponds to restrictions on the transitions between different maneuver states. This suggests more generally that we can associate particular hybrid state models with many practical algorithms which have been designed using intelligent heuristics and which are not based on any well-defined model. This association can help

clarify the assumptions on which a given practical algorithm is based and thus help to categorize different algorithms.

1.5 MULTIPLE FORMATIONS

We further demonstrate the power of the hybrid state approach by generalizing the multiobject tracking problem to include the problem of tracking multiple formations. Multiple formations arise naturally in surveillance problems and can be used to model such phenomena as aircraft flying in formation, aircraft launching missiles, and missiles intercepting aircraft to other missiles. The problem of tracking formations is generally more difficult than tracking individual targets because several returns in a single scan may be associated with one formation and because the dynamics of objects within a formation are correlated with each other.

The track-oriented data structure can be generalized to deal with multiple formations. This extension results in a three-level hierarchical structure consisting of global hypotheses at the highest level, local formation hypotheses at the intermediate level, and individual target hypotheses at the lowest level.

In the multiobject tracking algorithm without formations, the sufficient statistical information concerning discrete and continuous states can be computed and stored at the target local hypothesis (i.e., target track trees) level. This is no longer true in the formation tracking algorithm. In the formation tracking problem some discrete states (such as those indicating when targets separate or merge) are associated only with a formation and not with any individual target within a formation. Complications arise because the statistics of target continuous and discrete states depend on formation

hypothesis as well as the target hypotheses. Thus, the statistics of a specified target's continuous states and discrete states cannot be computed from knowing only the local target hypotheses. One also needs to know the local formation hypothesis.

These problems make the principal suboptimal methods difficult to apply to multiple formation problems. In conventional multiobject tracking one can apply screening at the target local hypothesis level during the creation of new target track nodes from measurement associations. In formation tracking this is not generally possible because the continuous state statistics necessary to define a measurement gate are computed and stored at the formation hypothesis level and not at the target level. Because targets may belong to more than one formation, there may be several measurement gates associated with each target track node. Thus, gating becomes more complicated. Difficulties also arise in pruning because the likelihood statistics used for pruning must be computed and stored at the formation level of the data structure and not at the target track level.

1.6 ORGANIZATION OF REPORT

This report is organized into two major technical sections and three appendices. Section 2 describes our approach to multiple maneuvering targets and some test cases of the algorithm we developed. Section 3 describes the abstract hybrid state model and shows how to treat multiple formations within this framework. It also describes the main features of the tracking algorithm needed for multiple formations. Section 4 concludes the report. Appendix A reviews several mathematical models of maneuvering targets, and Appendix B

reviews existing approaches to maneuver detection and tracking of single maneuvering targets. Appendix C describes the track-oriented approach and its relationship to other approaches in detail.

SECTION 2

MULTIOBJECT TRACKING OF MANEUVERING TARGETS

2.1 INTRODUCTION

For the multiobject tracking problem, if we assume that the dynamics of the individual targets and the measurement processes are linear and the noise processes are Gaussian, then the conditional density function of the state can be obtained by computing its sufficient statistics using a bank of Kalman filters. The essential difficulty for constructing the optimal algorithm then lies in forming all the possible discrete states and computing their probabilities. These discrete states correspond to the different possible combinations of targets with measurements. The combinatorial problem is especially severe since the algorithm has to also account for:

1. changes in number of targets due to births and deaths,
2. changes in dynamic models of targets due to maneuvers,* and
3. changes in measurement characteristics due to clutter or missed measurements.

Two approaches recommended in the past [2] have attempted to represent the hypotheses in the form of a matrix. In one of these approaches, referred

*There are several ways in which a maneuver can be modeled. Appendix A summarizes some of these models.

to as the target-oriented approach [3], the postulated targets define the columns of the matrix and the postulated hypotheses define the rows. The indices of the matrix represent measurements. Then for a given row (hypothesis), the column numbers and the measurements in the associated columns specify the target-measurement pair postulated by that hypothesis. A typical hypotheses matrix is shown in Fig. 2-1. The "0" entries indicate that the target is not detected.

TARGET NUMBER

HYPOTHESIS NUMBER	1	2	3
1	1	2	3
2	1	2	0
3	1	0	3
4	1	0	0
5	0	2	3
6	0	2	0
7	0	0	3
8	0	0	0

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Figure 2-1. Hypotheses Matrix for Target-Oriented Approach.

In the alternate approach, referred to as the "measurement-oriented approach," the roles of the targets and measurements are interchanged. A typical hypotheses matrix using this approach is shown in Fig. 2-2. Here the "0" entries denote that the measurements heading those columns are assumed to be false alarms.

		MEASUREMENT NUMBER 1	2	3
HYPOTHESIS NUMBER	1	1	2	3
	2	1	2	0
	3	1	0	3
	4	1	0	0
	5	0	2	3
	6	0	2	0
	7	0	0	3
	8	0	0	0

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Figure 2-2. Hypotheses Matrix for Measurement-Oriented Approach.

Both approaches have drawbacks. For example, in the target-oriented approach, measurements not included in a row could correspond to either new targets or false alarms; this cannot be shown explicitly. Similarly, in the measurement-oriented approach, targets not included in a row could have died or were not detected. The hypotheses matrix cannot display it. Furthermore, neither of the approaches can account for target maneuvers. The problem of maneuver detection has been studied mostly from the perspective of a single target. A survey of maneuver detection schemes for single targets is included in Appendix B.

2.2 TRACK-ORIENTED APPROACH FOR OPTIMAL HYPOTHESES REPRESENTATION

To overcome these problems, we have chosen to create the hypotheses at any scan in a novel fashion which is also intuitively appealing. Rather than representing the hypotheses in the form of a matrix, this approach maintains a

set of target trees and a list of global hypotheses. The root of each target tree represents the birth of the target and the branches represent the different dynamics that the target can assume and the various measurements it can be associated with in subsequent scans. A trace of successive branches from a leaf to the root of the tree corresponds to a potential track of the target. The leaf of each such trace is unique and is referred to as a track node of the target tree.

Each element of the global hypotheses list contains a set of pointers which point to track nodes. They represent the combination of track nodes postulated by the global hypothesis which that element represents. By assumption the collection of pointers in any one such global hypothesis cannot point to two track nodes within the same target tree. This implies that there is at most one return per target per scan.

The creation of the global hypotheses using target trees and global hypotheses list enables the decomposition of the process of associating targets with measurements into that of associating measurements with each of the targets and then forming combinations of the resulting tracks. As such, we refer to this as a track-oriented approach.* The expansion of the individual tracks at any scan can, in turn, be done in two stages. First the tracks are split for different possible dynamics and next these tracks are associated with the measurements. By assuming that the target dynamics are independent of the measurement characteristics, the transition diagrams for each of the targets and the measurements will have the simple form described below.

*The track-oriented approach and its relation to other approaches is described more fully in Appendix C.

The discrete states and the associated transition diagram for a single maneuvering target is considered first. The target starts off in an unborn state ($\overline{B}$), is born at some scan and can then die (X) at some later scan. A target that is in the born state can have a constant velocity (nonmaneuver state S) or be accelerating (maneuver state M). To allow for different accelerations that the target can undergo, there could be several maneuver states M_i ($i=1,n_m$). This is depicted in Fig. 2-3 where we have considered the case where $n_m=2$. For convenience in representing the transition diagrams, we have made the following assumptions:

1. the target is always born into the nonmaneuver state, and
2. the probability of transitioning to any of the born states or X is independent of the current born state of the target.

The transition diagram associated with the meaurement process is shown in Fig. 2-4. Observe that the probability of transitioning to either state is independent of the prior state. To prevent the existence of targets that have never been detected, we assume that a target that is born in the current scan will be detected. An alternate way of defining this requirement is to define the number of births parameter (in the distribution assumed for births) conditioned on the event that it will be detected. This also implies that the number of births conditioned on the event that it will not be detected is assumed to be zero.

Now we can depict the construction of the global hypotheses in any scan. As mentioned, the track nodes of all target trees are extended in two steps. In Step 1 the track node is split into several branches - to account for each of the several dynamic states that the target can be in. This is shown in

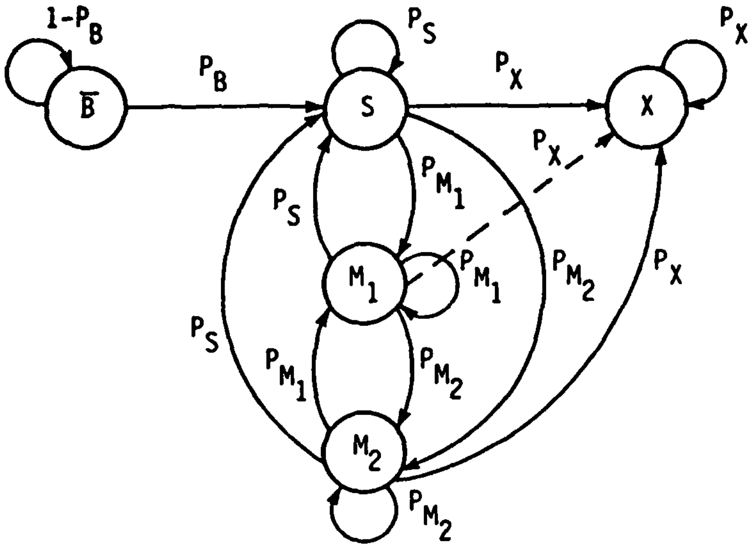

Figure 2-3. Transition Diagram for Target Dynamics States.

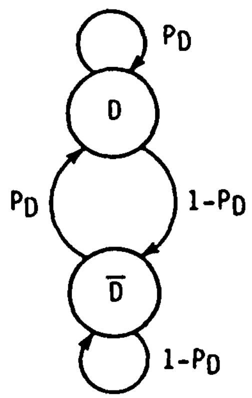

Figure 2-4. Transition Diagram for Measurement States.

Fig. 2-5. A parent track node that corresponds to a dead track is not split; only a continuation of the dead status is shown in this case.

In Step 2, the extended track nodes (excluding those that correspond to dead tracks) are associated with the measurements received in that scan. New track nodes are also generated to account for the possibility of a missed detection. Hence if there are n_r returns in the scan, then each of the track nodes will have $(1+n_r)$ descendents. We have extended the tree in Fig. 2-5 to illustrate the effect that Step 2 has on the track splitting process for the case where n_r=2 (Fig. 2-6). It is easy to see that for the general case, the number of track descendents for maneuvering targets is

$$[1 + (1+n_m)(1+n_r)] \quad . \qquad (2\text{-}1)$$

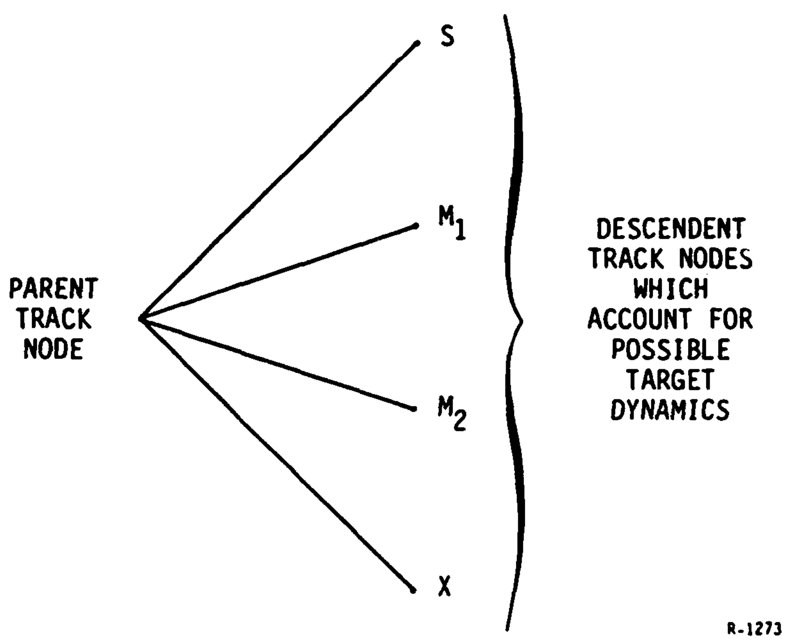

Figure 2-5. Track Splitting to Account for Different Target Dynamics.

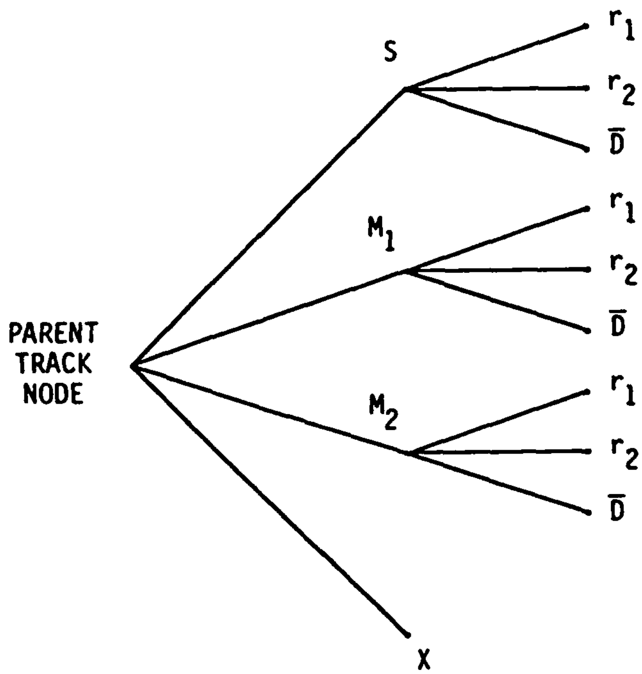

Figure 2-6. Track Splitting to Account for Different Target Dynamics and Different Measurement Associations.

Now we form all combinations of track nodes, which are descendents of parent track nodes included in the parent global hypothesis list, with the restriction that no two track nodes included in a new global hypothesis list should have the same parent node or use the same measurement from the current scan. We can show that for a parent global hypothesis which postulates the existence of n_t tracks, the number of descendent global hypotheses is

$$\sum_{i=0}^{\min(n_t,n_r)} \left(1 + \frac{n_m}{2}\right)^{n_t - i} (1+n_m)^i \; 2(n_r+n_t-2i) \; \frac{n_r!\,n_t!}{(n_t-i)!\,(n_r-i)!\,i!} \qquad (2\text{-}2)$$

where n_m = number of possible maneuvers, and

n_r = number of returns in current scan.

2.2.1 Likelihood Computation

The likelihood of any descendent global hypothesis has been shown to be

$$P[q_i^k|z^k] = \frac{1}{C_k}\,[P(q_i(k)|q_i^{k-1},z^{k-1})p(z(k)|q_i^k,z^{k-1})P(q_i^{k-1}|z^{k-1})] \qquad (2\text{-}3)$$

where k denotes scan number, z denotes measurements, q denotes discrete states, and the subscript i denotes the specific hypothesis. Since the likelihoods are used as a basis for comparing the various global hypotheses, we can ignore the denominator - it being the same for all. The first term in the numerator represents probabilities of transitioning from the parent global hypothesis to each of the descendent global hypotheses. Posterior likelihoods of these tracks after associating them with the different measurements available in the scan are represented by the second term. Finally, the last term is the likelihood of the parent global hypotheses.

If the likelihood of a false alarm is normalized to unity, the remaining measurement association likelihoods can be scaled accordingly. In such a case, we need only consider the likelihoods for the track nodes shown in Fig. 2-6 for each of the targets. This makes it possible to compute the likelihood of a descendent global hypothesis following the same steps used for constructing it.

The state transition diagram for the target dynamics (Fig. 2-5) defines the transition probabilities between different target states. The posterior likelihoods of the measurement associations can be obtained from a Kalman filter after being premultiplied by P_D, the probability of detection in one scan. The tracks which are postulated as being missed are multipled by $(1-P_D)$. Thus, the likelihoods of each of the descendent track nodes can be computed. Then,

after the proper descendent track nodes have been selected, the likelihood of the descendent global hypothesis can be computed as a product of the likelihood of the parent global hypothesis and the likelihoods of all the descendent track nodes included in it.

2.3 SUBOPTIMAL TECHNIQUES

The main purpose for designing suboptimal techniques is to reduce the computational burden associated with the optimal algorithm. Within the context of the optimal algorithm that we have constructed above, the computational burden can be reduced by either discarding some of the unlikely global hypotheses or using some computationally simpler algorithms for estimating the continuous-valued states. We will discuss only the former; several standard suboptimal techniques for the latter can be found in the literature (e.g., α-β tracker, constant gain Kalman filter).

Techniques available for reducing the number of global hypotheses can be grouped into one of the following:

1. screening,
2. pruning,
3. merging, or
4. clustering.

Both screening and pruning use the likelihoods to determine whether hypotheses should be discarded. Merging corresponds to the process of combining similar hypotheses. Grouping hypotheses in order to process the group as a unit independent of other groups is referred to as clustering. Since the optimal algorithm constructs global hypotheses in two stages, the hypotheses reducing techniques can be applied during either the track expansion stage or the global hypotheses building stage.

Screening techniques prevent less likely hypotheses from being formed or discard them after they are partially formed. We have incorporated several such options in the optimal algorithm. The first one is that of creating gates around track nodes and testing whether a measurement falls within this gate prior to forming a new descendent track node. Since not much screening can be achieved at the time of track initiation, an additional screening option has been provided. This is to prevent initiation of maneuvers in target dynamics until its track is "well established." By well established tracks we mean tracks for which the velocity uncertainty is below a certain threshold. This screening technique will prevent the inclusion of tracks which postulate maneuvers as a consequence of the large gate sizes at the time of track initiation. If a target were to maneuver at the time of birth, it will be picked up as a new target with little loss of information caused by dropping the previous track.

Two other screening options that we have introduced are based on the physical limitations of the target. One takes into account the finite velocities that a target can have; the initial uncertainty of velocity states can be chosen to reflect this. The other option takes into account the finite accelerations that are feasible for a target; we restrict the target from executing several different maneuvers in succession. In terms of the transition diagram shown in Fig. 2-3, this restriction implies that once a target enters a maneuver state, it can either remain in that state or return to the constant velocity state.

Pruning techniques discard hypotheses after they are formed. It can be effected in two ways: either deleting hypotheses which have a likelihood below a certain threshold or by limiting the global hypotheses at any stage to

a fixed number. The former is difficult to design since the threshold will, in general, be time varying. The latter technique is simpler to implement since there are no thresholds to be designed. We have incorporated the second option into our algorithm.

Gating is a powerful screening technique which drastically reduces the computational requirements with minimal increase in the probability of error. However, in the case where several targets are in close proximity for some period of time (crossing targets) or when the clutter density is high, then several measurements will fall within the gate giving rise to several descendant tracks.

To illustrate the problems that this could give rise to, let us consider the case of two targets that cross paths as shown in Fig. 2-7. For simplicity we have assumed that the probability of detection is 1 for both targets. At the time the targets cross, the measurement gates for either target will include the measurements from both targets. Hence, the tracks for both targets have to be split to accommodate the two measurements. Due to the close proximity of the measurements, the likelihoods for the split tracks will almost be the same. This will prevent the pruning algorithm from discarding either track so that the tracking algorithm will propagate the tracks with both associations.

Again, due to the close proximity of the measurements, the estimated states carried by the two tracks will have only small differences; in successive scans these differences become even smaller. By Scan 5, in Fig. 2-7, for example, the state estimates in the two tracks will become insensitive to the measurement association used in Scan 3. It might then be appropriate to either drop one of the tracks or combine them. This line of argument can be extended to the concept of dropping or combining global hypotheses.

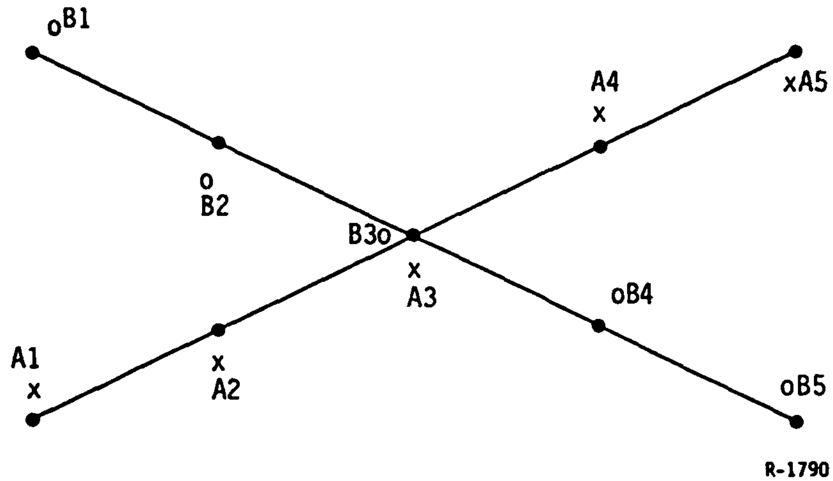

Figure 2-7. Measurements For Crossing Targets.

The general procedure for merging global hypotheses ([1],[4]) is to resolve target-measurement ambiguities in a particular scan n-scans later. This is referred to as the n-scan procedure. In this approach, however, defining and computing the similarity of tracks will not be straightforward. A computationally simpler approach is to resolve target-measurement ambiguities in a particular scan in the same scan. This is referred to as the zero-scan procedure. Obviously such a procedure will not have the benefit of using the measurements in later scans for resolving the ambiguity.

Again since the data structure used in our algorithm constructs the global hypotheses in two steps, our approach then has been to use a zero-scan pruning (or merging) technique at the track level. This implies that each track is associated with only a limited number (minimum one) of measurements lying within the measurement gate. Notice that the track in the previous scan would have already been split to accommodate maneuvers. Furthermore, there is no pruning (or merging) at the global hypotheses level indicating that at this

level it is an n-scan procedure. This retains the robustness of the algorithm. Since the heuristic is introduced at the track-measurement association level, it is most effective in reducing computational requirements when either the clutter density is large or when targets get close together.

2.4 SIMULATION RESULTS

Due to the large computational requirements of the optimal algorithm, it is not feasible to run any test scenario for more than 2 or 3 scans. Hence, we have run the algorithm with both screening and pruning options, discussed in subsection 3.2, in effect. Two test cases that were simulated are described below.

2.4.1 Test Case 1

We have considered the simple case of a single target having the trajectory shown (indicated by the continuous line) in Fig. 2-8. The target starts with a heading of 30°. At Scan 5, it executes a maneuver (-30° turn) and thereafter maintains a heading of 0°. We have generated clutter at every scan represented by squares). Figure 2-8 indicates the location of clutter at each scan.

We have summarized the simulation parameters in Table 2-1 and the tracking algorithm parameters in Table 2-2. The heuristics that have been used to reduce the computational requirements are given in Table 2-3. Using the measurement noise specified in Table 2-1, it can be shown that the uncertainty in the velocity estimates will be reduced to less than 10 m/sec after five scans. This ensures that the heuristic that initiates maneuver hypotheses only after tracks are well established, will postulate maneuvers for the target prior to the actual maneuver at Scan 5.

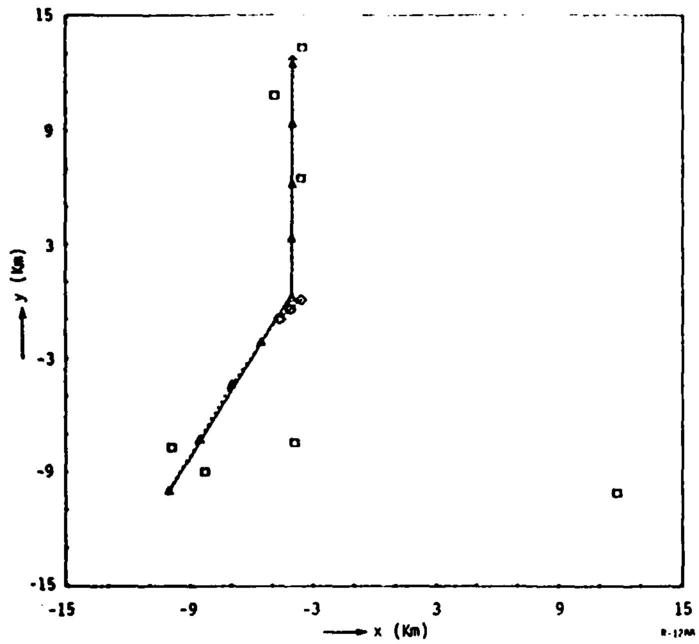

Figure 2-8. Test Case 1, Rank 1 Global Hypothesis.

TABLE 2-1. SIMULATION PARAMETERS

Scan time T: 10 sec

Number of scans: 9

Surveillance area: $-15{,}000 \text{ m} < x < 15{,}000 \text{ m}$
$-15{,}000 \text{ m} < y < 15{,}000 \text{ m}$

Measurement noise: $\sigma_x = 30$ m
$\sigma_y = 300$ m

Initial velocity of target: (Speed: 300 m/s, 30° heading)

Target velocity after fifth scan: (Speed: 300 m/s, 0° heading)

Clutter: 1 per scan

Uniform between

$(x_t(k)-1500)$ and $(x_t(k)+1500)$

$(y_t(k)-15{,}000)$ and $(y_t(k)+15{,}000)$

TABLE 2-2. PARAMETERS USED IN TRACKING ALGORITHM

Initial filter covariance: $\mathrm{Diag}[p_{11},p_{22},p_{33},p_{44}]$

p_{11},p_{22}: Set based on position measurement uncertainty

$p_{33}^{1/2} = p_{44}^{1/2} = 200$ m/sec

Model uncertainty: $\mathrm{Diag}[q_{11},q_{22},q_{33},q_{44}]$

$q_{11}^{1/2} = q_{22}^{1/2} = 0$

$q_{33}^{1/2} = q_{44}^{1/2} = 5$ m/sec

Measurement noise uncertainty

$\sigma_x = 30$ m

$\sigma_y = 300$ m

Dynamic model of target

$$\phi_m = \begin{matrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & a_m \cos\theta_m & a_m \sin\theta_m \\ 0 & 0 & -a_m \sin\theta_m & a_m \cos\theta_m \end{matrix}$$

$a_m \in \{1\}$

$\theta_m \in \{-30°, 0°, +30°\}$

Temporal distribution for births: Poisson with $\lambda_B = 10^{-5}$

Temporal distribution for false alarms: Poisson with $\lambda_{FA} = 4.5 \times 10^{-10}$

Probability of detection: 0.998

Probability of death: 2.0×10^{-4}

Probability of no maneuver: 0.8

Probability of maneuver: $0.2/n_m$

TABLE 2-3. HEURISTICS USED

1. Gating: 10
2. Number of global hypotheses retained at each scan: 10
3. Maximum number of missed detections permitted for a track: 2
4. Maneuver hypotheses initiated only for well-established tracks for which:

$$P_{33}^{1/2} < 15 \text{ m/sec}$$

$$P_{44}^{1/2} < 15 \text{ m/sec}$$

5. After maneuver is initiated, only transitions permitted are either straight-line or same maneuver state.
6. A Priori information about target position at birth is ignored, i.e., target position is initialized based on measurement data only.

$$P_{11}^{1/2} = \sigma_x \qquad x_1(0) = x_m$$

$$P_{22}^{1/2} = \sigma_y \qquad x_2(0) = y_m$$

Figure 2-8 also shows the trajectories postulated by the global hypothesis with the highest likelihood (highest rank). It can be seen that it identifies the correct target trajectory (dotted line through the triangles). However, it postulates the existence of another target (dotted line through the squares) that is born, detected, not detected, and dead in successive scans starting with the fourth scan. This is a consequence of the large gates associated with the targets that are just born. At Scan 9 such a track is insignificant and hence can be ignored.

We have shown in Figs. 2-9 and 2-10 the trajectories postulated by the global hypotheses with Ranks 2 and 3. It can be seen that the Rank 2 global

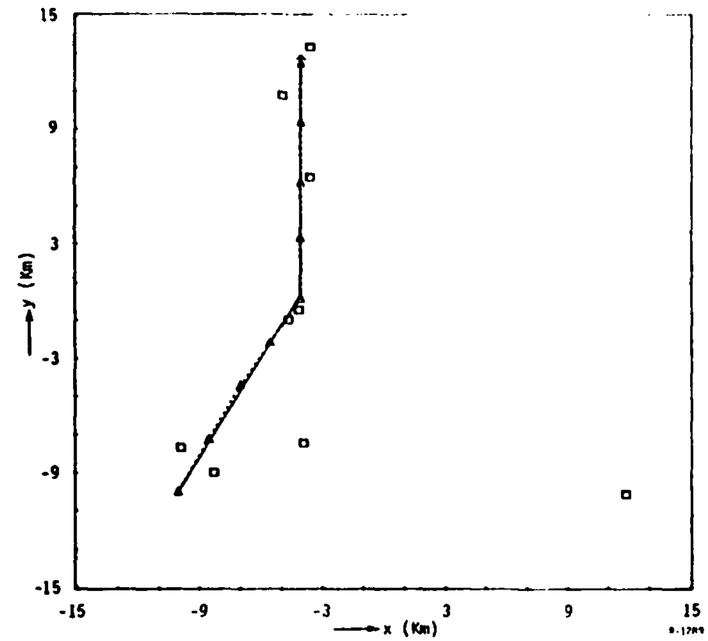

Figure 2-9. Test Case 1, Rank 2 Global Hypothesis.

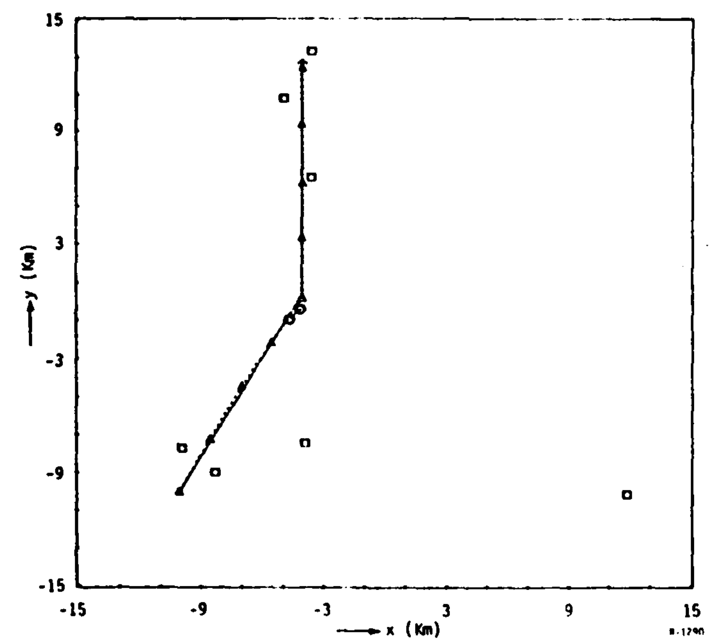

Figure 2-10. Test Case 1, Rank 3 Global Hypothesis.

hypothesis is the correct one - it postulates only the correct trajectory. The Rank 3 global hypothesis is almost identical to the Rank 1 hypothesis. The difference is that the incorrect track is postulated to die at Scan 6.

On examining the remaining global hypotheses which are retained by the algorithm (which are not shown here), we have observed they all postulate the correct trajectory for the target. Due to the modeling of the target dynamics in discrete time, however, some of them postulate a maneuver initiated at Scan 9. Since the position of the target will be influenced by a maneuver only in the next scan, it is only then that the algorithm will reject such incorrect hypotheses. As in the case of global hypotheses with Ranks 1 and 3, we have observed that some of the remaining global hypotheses postulate incorrect tracks for short periods of time. Since they are ephemeral, they do not have any adverse effect on the correct target trajectory. This feature of the algorithm, wherein most of the hypotheses that are retained postulate the correct trajectory with some minor differences, illustrates one aspect of the robustness of the suboptimal algorithm.

2.4.2 Test Case 2

In this test scenario, we simulated two crossing targets along with clutter. The targets cross at the same point in time. At that time one of the targets executes a maneuver too. As in the last scenario, clutter is generated close to targets. The true target trajectories are indicated by continuous lines and the clutter is indicated by squares. This is shown in Fig. 2-11. The parameters for the simulation and the algorithm are the same as in Test Case 1 with the following addition.

- Target 2 - Initial Position (-9657 m, -5264 m)

  - Speed and Heading (200 m/s, -45°)

It can be observed that the conditions are particularly severe for the tracking algorithm at the target crossing point where Target 1 executes the maneuver. Figure 2-11 traces the trajectories postulated by the global hypothesis with the highest likelihood (Rank 1). Despite the exacting requirements of the scenario, the algorithm identifies the correct hypothesis by the ninth scan. From these two test cases, we see that the algorithm performs very well in spite of the heuristics that have been introduced.

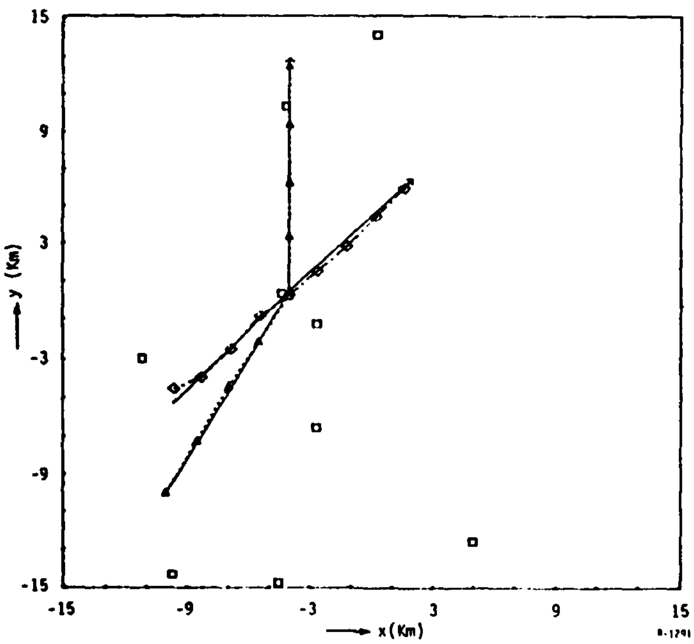

Figure 2-11. Test Case 2, Rank 1 Global Hypothesis.

2.5 MULTIOBJECT TRACKING WITH NONLINEAR RADAR MEASUREMENTS

Our simulation model hitherto provided rectangular position (x and y) measurements to the tracking algorithm. As such, a Kalman filter included in the tracking algorithm provided the optimum (minimum-variance) estimate of position and velocity of the targets.

In reality radars provide range and azimuth measurements of targets. This will require a nonlinear estimation algorithm in place of the linear (Kalman) filter to estimate the continuous-valued states of position and velocity for each target. Computationally feasible optimum nonlinear filters are not available. Suboptimal techniques use some form of linearization [5]-[7]. It has been shown that the extended Kalman filter, EKF, or (which uses the first two terms of a Taylor series expansion about the estimated state) has a performance close to the Cramer-Rao lower bound [8].

There are several choices for the state variables. If the position and velocity are expressed in polar coordinates, then the measurement equations are linear but the dynamic equations are nonlinear. The converse is true for the case where the states are chosen in the rectangular frame. Again, in [8], it has been shown that the filter with states in the polar frame is prone to divergence. Hence, we have chosen to implement the EKF in the rectangular frame.

It should be pointed out that depending on mission requirements, other suboptimal techniques such as an α-β tracking algorithm can be used. A complete discussion of such approaches and their shortcomings is given in [9]. The potential problems associated with limited precision of the computations on the EKF are also discussed there; here we assume that sufficient precision in the computations is available.

We have considered the case of two crossing targets having the trajectories indicated by the continuous line in Fig. 2-12. The location of clutter is indicated by squares. The parameters used in the simulation are summarized in Table 2-4.

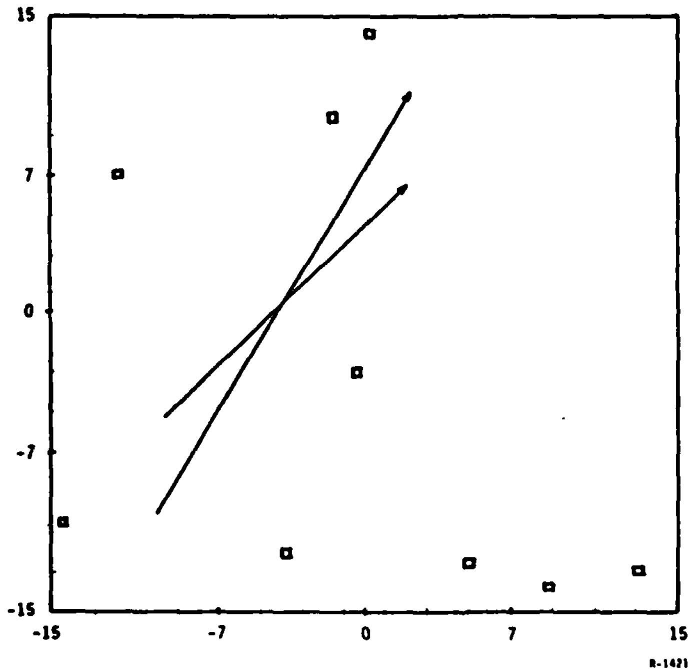

Figure 2-12. True Target Trajectories.

TABLE 2-4. SIMULATION PARAMETERS

Scan time T:	10 seconds
Number of scans:	9
Surveillance area:	$-15{,}000$ m $< x <$ 15,000 m $-15{,}000$ m $< y <$ 15,000 m
Measurement noise:	$\sigma_R = 8.2$ m $\sigma_\theta = 0.9$ deg
Target 1:	Initial position (-10,000 m, -10,000 m)
	Speed 300 m/s, Heading -30°
Target 2:	Initial position (-9657 m, -5264 m)
	Speed 200 m/s, Heading -45°
Clutter:	1 per scan, uniform density over surveillance area

The tracking algorithm uses an EKF to estimate the continuous-valued states of position and velocity. The parameters used by the filter are given in Table 2-5. Both screening and pruning options discussed earlier are used to cut down the computational requirements. These are summarized in Table 2-6.

The tracks postulated by the global hypothesis of highest rank are shown in Fig. 2-13. The broken line through the triangles represents the estimated track of Target 1 and the broken line through the diamonds represents the estimated track of Target 2. On examining the associations of the estimated tracks and the measurements (this cannot be seen in the figure), it is seen that they are incorrect at Scan 5. This is due to the fact that the targets cross at Scan 5 and so the returns from the two targets lie in close proximity. The incorrect association, however, does not alter the estimated tracks from the ones obtained with the right associations.

TABLE 2-5. FILTER PARAMETERS

Initial filter covariance:
$$p(0) = \begin{bmatrix} P_p & 0 \\ 0 & P_v \end{bmatrix}$$
Position uncertainty matrix P_p: Set based on Option B of heuristics
Velocity uncertainty matrix P_v: $\mathrm{Diag}[P_{33}, P_{44}]$
$$p_{33}^{1/2} = p_{33}^{1/2} = 200 \text{ m/sec}$$
Model uncertainty: $\mathrm{Diag}[g_{11}, g_{22}, g_{33}, g_{44}]$
$$g_{11}^{1/2} = g_{22}^{1/2} = 0$$
$$g_{33}^{1/2} = g_{44}^{1/2} = 5 \text{ m/sec}$$
Measurement noise uncertainty:
Range: $\sigma_R = 8.2$ m
Bearing: $\sigma_\theta = 0.7$ deg

TABLE 2-6. HEURISTICS USED

1. Gating: 3
2. Number of global hypothesis retained at each scan: 10
3. Maximum number of missed detections permitted for a track: 2
4. Target position is initialized based on measurement data only:

$$x_1(0) = R_m \cos\theta_m \qquad x_2(0) = R_m \sin\theta_m$$

$(R_m, \theta_m$ = measured range and bearing)

$$P_p = \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix}$$

$$p_{11} = \sigma_R^2 \cos\theta_m + R_m^2\sigma_\theta^2 \sin^2\theta_m$$

$$p_{22} = \sigma_R^2 \sin\theta_m + R_m^2\sigma_\theta^2 \cos^2\theta_m$$

$$p_{12} = \cos\theta_m \sin^2\theta_m(\sigma_R^2 - R_m^2\sigma_\theta^2)$$

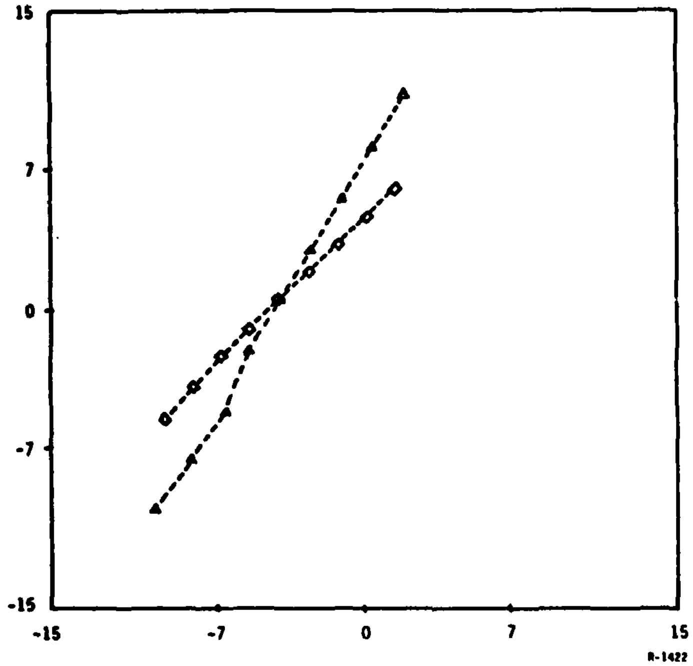

Figure 2-13. Estimated Tracks in Rank 1 Global Hypothesis.

The global hypothesis of Rank 2 postulates the right associations. It is not shown here since it looks identical to the hypothesis of Rank 1. We can eliminate one of these two hypotheses, thereby reducing the computational requirements. This has been verified in a simulation run.

In this section we will consider a generalization of the multiobject tracking problems considered in Section 2 and in our earlier works [12]. The problem of tracking multiple formations arises in many realistic surveillance missions. This section will demonstrate the power of the hybrid state approach by formulating such problems in concise mathematical terms and analyzing the structure of the resulting tracking algorithm.

SECTION 3

MULTIOBJECT TRACKING OF FORMATIONS

3.1 INTRODUCTION

In this section we will consider a generalization of the multiobject tracking problems considered in Section 2 and in our earlier work [12]. The problem of tracking multiple formations arises in many realistic surveillance missions. This section will demonstrate the power of the hybrid state approach by formulating such problems in concise mathematical terms and analyzing the structure of the resulting tracking algorithm. We will use the term formations generally to refer to multiple targets whose dynamics are correlated. This term arises from the particular example in which several aircraft purposely fly in close proximity to each other, i.e., fly in a formation. We also use the term formations to describe more general situations of correlated motion such as when one target splits into several targets (e.g., when an aircraft launches a missile), or when several targets merge into one target (e.g., when one missile intercepts another). The essential feature of each of these examples is that the motions of several different objects are dependent on other. One can use knowledge of these dependences to improve tracking similarly to the way one uses knowledge of maneuvers to improve tracking. Conversely, one can use knowledge of target motion estimates to detect dependences (e.g., detect missile launches or verify interception) similarly to the way one uses target motion estimates to detect maneuvers. In either case, we

have a coupled detection-estimation problem analogous to the maneuvering target problem we discussed earlier.

Conceptually, it is convenient to think of a group of correlated targets as a single entity, which we will call a formation. Thus, the overall surveillance environment consists of a number of formations, and each formation contains a number of targets. Note that a formation may contain only one target in some cases.

From this point of view, different formations have independent dynamics, and these dynamics can be efficiently modeled by independent hybrid state systems. In this respect, the modeling and treatment of a single formation is completely analogous to the modeling and treatment of a single maneuvering target. What makes tracking formations a more difficult problem is that the independent dynamic objects (that is, each formation) can have more than one return associated with them in each scan period. In addition, the number of targets within a formation may vary with time. Thus, each formation presents a multiobject tracking problem in microcosm.

This section is organized as follows. In subsection 5.2 we formulate formation tracking in terms of hybrid state systems. We do this both for the general formation tracking problem and for several specific examples. Subsection 5.3 describes the optimal Bayesian algorithm for solving the formation tracking problem; it also discusses suboptimal approximations required to implement a practical tracking algorithm.

3.2 HYBRID STATE MODEL OF FORMATION TRACKING

We have found it useful to view multiobject tracking in terms of a partially observed Markov chain whose state and observation each consist of a

continuous part and a discrete part [1], [10]-[14]. In this subsection we briefly describe this approach in general and then show how it specializes to the problem of tracking formations.

3.2.1 Hybrid State Systems

Let x be the continuous component of the Markov state process and let q be the discrete component. The joint state process $s(t)=(x(t),q(t))$ is Markov in the usual sense that given s(t), the probability distribution of s(t+1) is independent of the past process s(t-1), s(t-2), etc. We denote the probability of transition from $x(t)=x_1$, $q(t)=q_1$, to $x(t+1)=x_2$, $q(t+1)=q_2$ by the expression

$$p(x_2|q_2,x_1,q_1)P(q_2|x_1,q_1) \quad . \tag{3-1}$$

Because x is continuous-valued, the continuous transition probability p of Eq. 3-1 is in fact a probability density in x_2. The discrete transition probability of P is an honest probability in q_2. In our work we have assumed that p is a Gaussian density in the variable x_2 with mean m and covariance Σ given by

$$m = A(q_2|q_1)x_1 + b(q_2|q_1) \quad , \tag{3-2}$$

$$\Sigma = B(q_2|q_1)Q(q_2|q_1)B(q_2|q_1)^T \quad , \tag{3-3}$$

where T denotes the matrix transposition. If the continuous state has dimension n, then $A(q_2|q_1)$ is an nxn matrix, and the vector $b(q_2|q_1)$ has dimension n also. We assume that $Q(q_2|q_1)$ is a kxk positive definite matrix. In this case $B(q_2|q_1)$ is a matrix of dimension nxk. Note that one could also model time dependent n and k by putting this dimensional information in the discrete state. This could be used to model a changing number of targets. In the next subsection we will show how to model a changing number of targets using fixed dimensions.

The observations in a hybrid state problem consists of a continuous part y and a discrete part z. The joint probability distribution of y(t) and z(t) depends only on x(t) and q(t). This probability distribution is given by an expression similar to Eq. 3-1, namely

$$h(y_1|x_1,q_1) \cdot H(z_1|x_1,q_1) \tag{3-4}$$

which is the probability (density) of observing $y(t)=y_1$ and $z(t)=z_1$ given $x(t)=x_1$ and $q(t)=q_1$. As with p, we will assume that h is a Gaussian density in y_1 with mean m and covariance Σ given by

$$m = C(q_1)x_1 + d(x_1,q_1) \quad , \tag{3-5}$$

$$\Sigma = R(q_1) \tag{3-6}$$

where $R(q_1)$ is positive semidefinite. If the continuous observation has dimension j, then $C(q_1)$ is a jxn matrix, $R(q_1)$ is jxj, and $d(x_1,q_1)$ is a j dimensional vector. Note that $d(x_1,q_1)$ depends on x_1 only if we wish to consider nonlinear measurements (such as radar range and azimuth measurements treated earlier).

We can express the continuous state transition model and the continuous observation model more transparently in terms of the following stochastic difference equations. The continuous state transition density described by Eqs. 3-2 and 3-3 can be described by

$$x(t+1) = A(q(t+1)|q(t))x(t) + B(q(t+1)|q(t))w(q(t+1)|q(t),t) + b(q(t+1)|q(t)) \tag{3-7}$$

where $w(q_2|q_1,t)$ is a zero-mean, Gaussian random vector with covariance $Q(q_2|q_1)$, and the random vectors $w(q_2|q_1,t)$ are independent for different t. Similarly, the continuous observation density described by Eqs. 3-5 and 3-6 can be described by

$$y(t) = C(q(t))x(t) + d(x(t),q(t)) + v(q(t),t) \qquad (3\text{-}8)$$

where $v(q_1,t)$ is a zero-mean Gaussian random vector of covariance $R(q_1)$, and the random vectors $v(q_1,t)$ are independent for different t.

3.2.2 Features of Multiobject Tracking Models

In the previous subsection we described a general hybrid state system capable of modeling most multiobject tracking situations, including formations. In this section we discuss some of the structural features which the multiobject tracking problem imposes on this general hybrid state model. In particular, we will discuss how multiple hybrid state subsystems are combined into one hybrid state systems in order to model multiple targets or more generally, multiple formations. We also discuss how the multiobject tracking problem is reflected in the structure of the continuous state x(t) and the discrete state q(t).

Let us now develop a hybrid state model of N multiple independent dynamic objects (targets or formations). Let k=1,2,...,N denote a unique object label (i.e., object k). Model each dynamic object k using a hybrid state model of the type described in the previous subsection. Let $x_k(t)$, $q_k(t)$, $y_k(t)$, and $z_k(t)$ denote the continuous and discrete states, and the continuous and discrete observations associated with object k.

The appearance and disappearance of different objects is determined by the discrete states $q_k(t)$. That is, there is a special discrete state $\overline{B}$ (not born [12]) such that $q_k(t)=\overline{B}$ means that object k has not appeared up to time t, and there is a special discrete state X (dead [12]) such that $q_k(t)=X$ means that object k has disappeared by time t. By adjusting N, the individual transition probabilities from $\overline{B}$, and the transition probabilities to X, one can model the random inflow and outflow of objects and the average number of targets present in a surveillance region (see [12], Section 2 for further details). However, note that the maximum number of targets appearing at any time is bounded by N. Thus, the continuous dimension of the hybrid state model of the combined system will be finite. If N is large, this dimension will also be large, but this does not necessarily imply a large dimensional implementation of the tracking algorithm. The dimensionality of the implemented algorithm depends on the number of targets that have actually appeared up to a given time rather than on N.

The continuous state x(t) for the combined system of multiple objects is simply the Cartesian product of the N possible individual states $x_k(t)$. We write

$$x(t) = (x_1(t),\ x_2(t),\ \ldots,\ x_n(t)) \qquad (3\text{-}9)$$

to express this. That is, $x_k(t)$ is the k-th component of x(t). However, note that each component may itself be a vector.

The discrete state q(t) for the combined system consists of two parts. One part $q^1(t)$ is the Cartesian product of the individual $q_k(t)$, namely

$$q^1(t) = (q_1(t), q_2(t), \ldots, q_n(t)) \quad . \tag{3-10}$$

The other part $q^2(t)$ describes how received measurements are randomly associated with targets. We will define $q^2(t)$ below. The discrete state $q(t)$ is then the Cartesian product

$$q(t) = q^1(t) \times q^2(t) \quad . \tag{3-11}$$

Let $\tilde{y}(t)$ and $\tilde{z}(t)$ denote the continuous and discrete observations of the combined system. These are related to the individual measurements $y_k(t)$ and $z_k(t)$ in the following way. Define $y(t)$ and $z(t)$ as

$$y(t) = (y_1(t), y_2(t), \ldots, y_n(t)) \quad . \tag{3-12}$$

$$z(t) = (z_1(t), z_2(t), \ldots, z_n(t)) \quad . \tag{3-13}$$

Then $\tilde{y}(t)$ and $\tilde{z}(t)$ are given by

$$\tilde{y}(t) = \pi_c(q^2(t))y(t) \tag{3-14}$$

$$\tilde{z}(t) = \pi_d(q^2(t))z(t) \tag{3-15}$$

where $\pi_c(q^2)$ and $\pi_d(q^2)$ are permutation matrices. That is, for each value q^2 of $q^2(t)$, the matrices $\pi_c(q^2)$ and $\pi_d(q^2)$ consist of all 1's and 0's, and each row and each column contain exactly one 1. For each different value of $q^2(t)$, the permutation matrices represent different associations of received measurements with individual measurements $y_k(t), z_k(t)$, and hence represent

associations of received measurements with objects k. If each object k can give at most N_k returns in one scan period t, then there are

$$M = \left(\sum_{k=1}^{N} N_k \right) ! \tag{3-16}$$

possible different associations, and hence M possible values of q^2. We assume each value is equiprobable so that

$$\Pr\{q^2(t)=q^2\} = \frac{1}{M} \quad . \tag{3-17}$$

We also assume that $q^2(t)$ is a process of independent discrete-valued random variables which are also independent of $q^1(t)$. This completes our description of the discrete state q(t) of the combined system of multiple targets. We have also described the continuous state and the discrete and continuous measurements of the combined system.

It is not hard to see that x(t), q(t), $\tilde{y}(t)$, and $\tilde{z}(t)$ satisfy hybrid state system equations of the type we described in the previous subsection. Thus, we have shown how to model multiple objects (targets or more general objects such as formations) in terms of a combination of individual hybrid state systems modeling each object. In the next subsection we give examples of such models for different types of formations.

3.2.3 Examples of Formations

In the first two subsections we described a general hybrid state model of multiple, independent dynamic objects. In this subsection we will describe

specific models of different types of formations. These models give the hybrid state subsystems that make up the overall hybrid state system model for a multiobject tracking problem.

EXAMPLE 1: FLYING IN FORMATION

The simplest example of formations consists of a number of targets flying parallel to each other in close proximity i.e., flying in formation. We will assume that the targets are too close to resolve accurately, but that multiple returns in the form of position measurements are possible. The continuous states are the position and velocity of the center of the formation. The discrete states model the number of targets and the number of radar returns from the formation in each scan period. The continuous observations model the scatter of target position measurements around the formation center. The continuous state model is given by

$$x(t+1) = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} w(t) \tag{3-18}$$

where x(t) is the four-dimensional vector of formation center position and velocity, and w(t) is a two-dimensional zero-mean Gaussian white noise with covariance

$$Q = \begin{bmatrix} \sigma_w^2 & 0 \\ 0 & \sigma_w^2 \end{bmatrix} . \tag{3-19}$$

Note that in this simple model the continuous state does not depend on the discrete state. The discrete state $q(t)$ consists of two components, $q_1(t)$ and $q_2(t)$. The first component $q_1(t)$ is constant for all t and gives the number of targets in the formation. The second component $q_2(t)=1,2,\ldots,q_1(t)$ indicates how many returns are reflected by the formation. We assume that the process $q_2(t)$ is independent for different t and binomially distributed as

$$\Pr\{q_2(t)=k|q_1(t)\} = \binom{q_1(t)}{k} P_D^k(1-P_D)^{q_1(t)-k} \tag{3-20}$$

where P_D is the probability that one target of the formation will give a return in any one scan period.

The continuous observations for this example can be modeled either as a variable dimension observation vector or fixed (large) dimension observation vector. We will describe both here to illustrate the two approaches. In the variable dimension approach $y(t)$ is given by

$$y(t) = C(q_2(t))x(t) + v(q_2(t),t) \tag{3-21}$$

where $C(q_2)$ is the $2q_2$ x four-dimensional matrix

$$C(q_2) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix} . \tag{3-22}$$

The zero-mean Gaussian white noise $v(q_2,t)$ has dimension $2q_2$ and a $2q_2 \times 2q_2$-dimensional covariance of the block diagonal form

$$R(q_2) = \mathrm{diag}[R_V, R_V, \ldots] \tag{3-23}$$

where R_V is the 2x2 diagonal block.

The fixed dimensional observation model assumes a maximum possible number N of targets in the formation. Thus, $q_1(t)$ and $q_2(t)$ are bounded above by N. Equation 3-21 still holds, but $C(q_2)$ is a 2N x four-dimensional matrix of the form

$$C(q_2) = \begin{array}{c} \left[\begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \hline 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{array}\right] \end{array} \begin{array}{l} \left.\vphantom{\begin{array}{c}1\\1\\1\\1\end{array}}\right\} 2q_2 \\ \left.\vphantom{\begin{array}{c}1\\1\\1\\1\end{array}}\right\} 2N-2q_2 \end{array} \quad . \tag{3-24}$$

The zero-mean Gaussian white noise $v(q_2,t)$ has dimension 2N and a 2Nx2N-dimensional covariance matrix of the block diagonal form

$$R(q_2) = \mathrm{diag}[\overbrace{R_V, R_V, \ldots, R_V}^{2q_2} \mid \overbrace{0, 0, \ldots, 0}^{2N-2q_2}] \quad . \tag{3-25}$$

In this second approach 0 components in the observation vector y(t) signal the absence of any return. Note that in actual implementation it would not be necessary to deal with the entire 2N-dimensional matrices $C(q_2)$ and $R(q_2)$, but just the nonzero parts.

Note that we have omitted the discrete state that randomizes the association of returns as they are actually received. In the examples of this

subsection the models will only describe the subsystems discussed in subsection 3.2.2. Randomization of associations is carried out at the combined system level as discussed also in subsection 3.2.2.

Finally, note that if $\sigma_w=0$ and the velocity component of x(t) is 0, then this model is the same as our model of clutter distributed around the static position given by the first two components of x(t). Thus, in this example we have modeled a flying formation as pseudo-clutter tightly distributed around a constant velocity center.

EXAMPLE 2: MISSILE LAUNCH

The second example of formations is the case of a missile-carrying aircraft which may launch its missiles at an unpredictable time. For simplicity we will consider the case of an aircraft carrying a single missile. The continuous states are the positions and velocities of the aircraft and its missile. The discrete states model the random launch time. The continuous observations are the position of the aircraft and (after launch) the position of the missile.

The discrete state model is the simple two-state Markov chain shown in Fig. 3-1. We denote the states L (launched) and $\overline{L}$ not launched). The transition probabilities are defined in terms of the probability that the missile remains unlaunched, i.e., the probability $P(\overline{L}|\overline{L})$ of the transition from $\overline{L}$ to $\overline{L}$ as shown in Fig. 3-1. This simple discrete state model gives a geometrically distributed random launch time. The continuous state for this example obeys a linear Gaussian hybrid state difference equation such as Eq. 3-7 where A, B, and Q are given as follows.

$$A(\bar{L}|\bar{L}) = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3-26}$$

$$A(L|\bar{L}) = \left[\begin{array}{cccc} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \hdashline 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha\cos\theta & \alpha\sin\theta \\ 0 & 0 & -\alpha\sin\theta & \alpha\cos\theta \end{array}\right] \tag{3-27}$$

$$A(L|L) = \left[\begin{array}{cccc:cccc} 1 & 0 & T & 0 & & & & \\ 0 & 1 & 0 & T & & & & \\ 0 & 0 & 1 & 0 & & & & \\ 0 & 0 & 0 & 1 & & & & \\ \hdashline & & & & 1 & 0 & T & 0 \\ & & & & 0 & 1 & 0 & T \\ & & & & 0 & 0 & 1 & 0 \\ & & & & 0 & 0 & 0 & 1 \end{array}\right] . \tag{3-28}$$

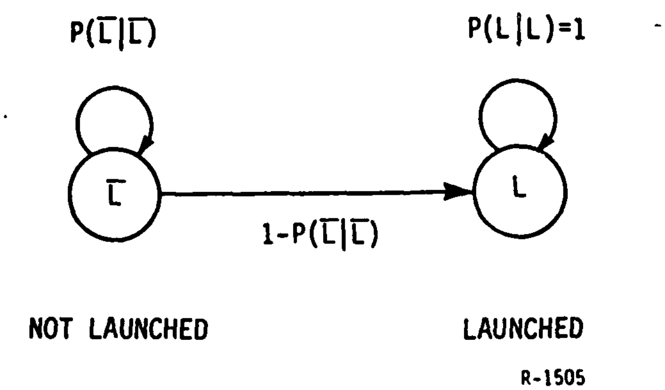

Figure 3-1. Discrete State Transitions for Missile Launch Example.

The blank spaces in $A(L|L)$ are filled with zeros.

$$B(\overline{L}|\overline{L}) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{3-29}$$

$$B(L|\overline{L}) = \left[\begin{array}{cc|cc} 0 & 0 & & \\ 0 & 0 & & \\ 1 & 0 & & \\ 0 & 1 & & \\ \hline & & 0 & 0 \\ & & 0 & 0 \\ & & 1 & 0 \\ & & 0 & 1 \end{array}\right] \tag{3-30}$$

$$B(L|L) = \left[\begin{array}{cc|cc} 0 & 0 & & \\ 0 & 0 & & \\ 1 & 0 & & \\ 0 & 1 & & \\ \hline & & 0 & 0 \\ & & 0 & 0 \\ & & 1 & 0 \\ & & 0 & 1 \end{array}\right] . \tag{3-31}$$

The covariance matrices $Q(\overline{L}|\overline{L})$, $Q(L|\overline{L})$, and $Q(L|L)$ are all diagonal matrices of dimensions 2x2, 6x6, and 4x4, respectively.

$$Q(\overline{L}|\overline{L}) = \text{diag}[\sigma^2_{w_1}, \sigma^2_{w_1}] \tag{3-32}$$

$$Q(L|\overline{L}) = \text{diag}[\sigma^2_{w_1}, \sigma^2_{w_1}, \sigma^2_{w_2}, \sigma^2_{w_3}] \tag{3-33}$$

$$Q(L|L) = \text{diag}[\sigma^2_{w_1}, \sigma^2_{w_1}, \sigma^2_{w_4}, \sigma^2_{w_4}] . \tag{3-34}$$

These matrices give a continuous state model with the following interpretation. While $q(t)=\overline{L}$, the continuous state $x(t)$ has four dimensions which model the aircraft's state according to constant velocity flight. When $q(t)$ first equals L (so $q(t-1)=\overline{L}$), then the continuous state $x(t)$ becomes eight-dimensional in order to model the aircraft and missile states. The transition matrix $A(L|\overline{L})$ in Eq. 3-27 gives the missile an initial position equal to the aircraft's current position, and it gives the missile an initial course which turned θ radians from the aircraft's course and an initial speed equal to a constant α times the aircraft's speed. In addition, the missile's velocity is given a Gaussian random component with covariance given by the diagonal matrix

$$\mathrm{diag}[\sigma^2_{w_2}, \sigma^2_{w_3}] \quad . \tag{3-35}$$

We assume here that α, θ are known. One could also consider possible random choices of α and θ by increasing the discrete state just as we have done to model maneuvering targets in our earlier work.

The continuous observations are modeled by Eq. 3-8 where

$$C(\overline{L}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{3-36}$$

$$C(L) = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & & & & \\ 0 & 1 & 0 & 0 & & & & \\ \hline & & & & 1 & 0 & 0 & 0 \\ & & & & 0 & 1 & 0 & 0 \end{array}\right] \tag{3-37}$$

and where R is diagonal as given by

$$R(\overline{L}) = \mathrm{diag}[\sigma^2_{v_1}, \sigma^2_{v_1}] \tag{3-38}$$

$$R(L) = \mathrm{diag}[\sigma^2_{v_1}, \sigma^2_{v_1}, \sigma^2_{v_2}, \sigma^2_{v_2}] \quad . \tag{3-39}$$

Note that we have only modeled this example using variable dimension continuous states and observations. One can also used a fixed dimension approach as in Example 1. furthermore, we have not modeled missed detections in this example to clarify the presentation. One could easily include this phenomenon by augmenting the discrete state as we did in our earlier work on maneuvers [14] and in[12].

EXAMPLE 3: MISSILE INTERCEPTION

The last example we will present is the case of one missile intercepting another. The two targets in this example of a formation interact (i.e., collide) if the targets are sufficiently close to each other. This interaction results in one or both targets being destroyed according to a probabilistic law (i.e., probability of kill).

There are four discrete states in this example: $\bar{I}$ (no interception), I (interception), K (kill), and $\bar{K}$ (no kill). The transitions and transition probabilities are shown in Fig. 3-2. Note that the transitions from $\bar{I}$ to $\bar{I}$ and from $\bar{I}$ to I depend on the continuous state x. This is necessary in order to model the dependence of the interception probability on the proximity of the two targets. We will describe this dependence below after we discuss the continuous state model. Note that $P(K|I)$ is the probability of a kill given that there was an interception.

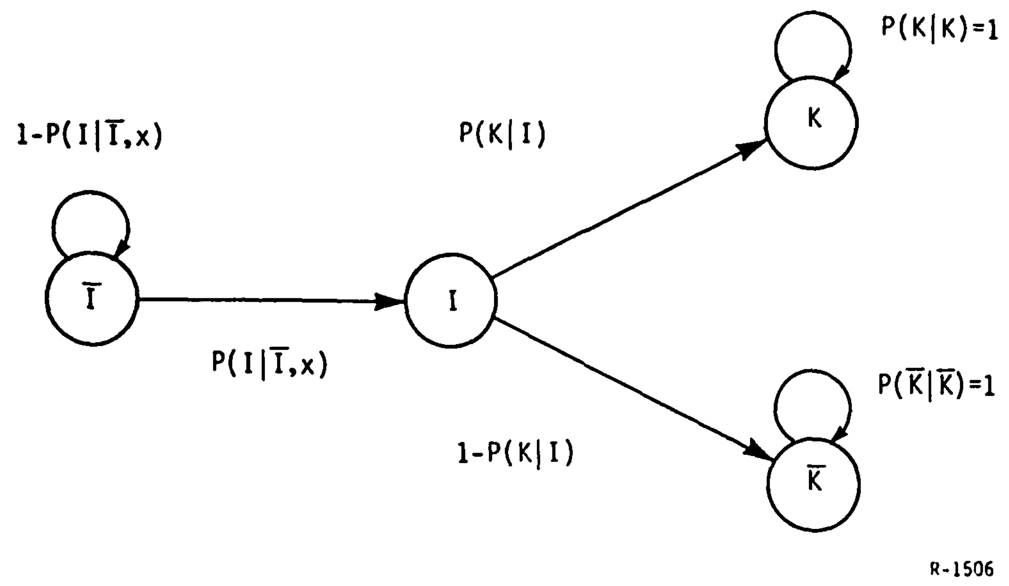

Figure 3-2. Discrete State Transitions for Missile Interception Example.

The continuous state model is given by the hybrid state equation (Eq. 3-7) where the different matrices take on values we give below. The discrete transition $\bar{I}$ to $\bar{I}$ corresponds to the two targets flying independently of each other. Thus, we have

$$A(\bar{I}|\bar{I}) = \left[\begin{array}{cccc|cccc} 1 & 0 & T & 0 & & & & \\ 0 & 1 & 0 & T & & & & \\ 0 & 0 & 1 & 0 & & & & \\ 0 & 0 & 0 & 1 & & & & \\ \hline & & & & 1 & 0 & T & 0 \\ & & & & 0 & 1 & 0 & T \\ & & & & 0 & 0 & 1 & 0 \\ & & & & 0 & 0 & 0 & 1 \end{array}\right] \tag{3-40}$$

$$B(\bar{I}|\bar{I}) = \left[\begin{array}{cc|cc} 0 & 0 & & \\ 0 & 0 & & \\ 1 & 0 & & \\ 0 & 1 & & \\ \hline & & 0 & 0 \\ & & 0 & 0 \\ & & 1 & 0 \\ & & 0 & 1 \end{array}\right] \qquad (3\text{-}41)$$

$$Q(\bar{I}|\bar{I}) = \mathrm{diag}[\sigma^2_{w_1}, \sigma^2_{w_1}, \sigma^2_{w_2}, \sigma^2_{w_2}] \quad . \qquad (3\text{-}42)$$

If the targets intercept, so that there is a transition from $\bar{I}$ to I, then we assume that one target (the interceptor missile) is destroyed, and the other target (the intercepted missile) is not destroyed unless the discrete state transitions to K. Thus, we have

$$A(I|\bar{I}) = \left[\begin{array}{cccc|cccc} 1 & 0 & T & 0 & & & & \\ 0 & 1 & 0 & T & & & & \\ 0 & 0 & 1 & 0 & & & & \\ 0 & 0 & 0 & 1 & & & & \end{array}\right] \qquad (3\text{-}43)$$

$$B(I|\bar{I}) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad (3\text{-}44)$$

$$Q(I|\bar{I}) = \mathrm{diag}[\sigma^2_{w_1}, \sigma^2_{w_1}] \quad . \qquad (3\text{-}45)$$

Note that the continuous state dimension decreases from eight to four when the discrete state transitions from $\bar{I}$ to I. In our example the probability of this transition depends on the eight-dimensional continuous state x. In our

representation, x_1 and x_2 are the two position coordinates of the intercepted missile and x_5 and x_6 are the position coordinates of the interceptor. The transition probability $P(I|\overline{I},x)$ should be chosen to be a function of the distance

$$d = [(x_1-x_5)^2+(x_2-x_6)^2]^{1/2} \tag{3-46}$$

so that $P(I|\overline{I},x)=0$ if d is greater than some interception distance d_0 and so that $P(I|\overline{I},x)=1$ if $d<d_0$.

If the discrete state is ever I, it will become either K or $\overline{K}$ in the next time period. The transition I to $\overline{K}$ implies that the intercepted missile was not killed, and in this case the continuous state model remains unchanged. That is, we have

$$A(\overline{K},I) = A(I|\overline{I}) \tag{3-47}$$

$$B(\overline{K}|I) = B(I|\overline{I}) \tag{3-48}$$

$$Q(\overline{K}|I) = Q(I|\overline{I}) \quad . \tag{3-49}$$

Note that we could identify $\overline{K}$ with I, or we could eliminate I and allow transitions directly from $\overline{I}$ to K or $\overline{K}$. We have included the redundant discrete state in order to make our model somewhat clearer.

Finally, if the discrete state transitions from I to K, then the intercepted missile is destroyed and the continuous state dimension goes from 4 to 0. That is, K plays the same role as the death state, X, we used to model target disappearance in our earlier work [12]. For this transition there is no need to specify $A(K|I)$, $B(K|I)$, or $Q(K|I)$.

The continuous observation models corresponding to the discrete states I, $\overline{I}$, K, $\overline{K}$ are given by C, R, and Eq. 3-8. In discrete state I there are two targets observed, in state $\overline{I}$ and $\overline{K}$ only the intercepted target is observed. Thus, we have

$$C(\overline{I}) = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & & & & \\ 0 & 1 & 0 & 0 & & & & \\ \hline & & & & 1 & 0 & 0 & 0 \\ & & & & 0 & 1 & 0 & 0 \end{array}\right] \tag{3-50}$$

$$C(I) = C(\overline{K}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{3-51}$$

and we need not specify C(K) since nothing is observed. Similarly, we have

$$R(\overline{I}) = \mathrm{diag}[\sigma^2_{v_1}, \sigma^2_{v_1}, \sigma^2_{v2}, \sigma^2_{v_2}] \tag{3-52}$$

$$R(I) = R(\overline{K}) = \mathrm{diag}[\sigma^2_{v_1}, \sigma^2_{v_1}] \tag{3-53}$$

and we need not specify R(K).

At this point several comments are in order. We have presented the simplest hybrid state model of missile interception. More complex and more realistic models are possible. For example, one easily includes missed detections as before. An additional feature of more practical interest is to expand the discrete state K to model a clutter formation centered at the interception position. This formation disappears in a few time periods after interception, and it can be used to model the observed wreckage of a successfully intercepted missile. In the example presented above, the two missiles are modeled as completely independent moving objects before interception.

However, it is more realistic to model the interceptor missile as following the intercepted missile. This feature can be added by changing $A(\overline{I}|\overline{I})$ in Eq. 2-40 to something like

$$A(\overline{I}|\overline{I}) = \left[\begin{array}{cccc|cccc} 1 & 0 & T & 0 & & & & \\ 0 & 1 & 0 & T & & & & \\ 0 & 0 & 1 & 0 & & & & \\ 0 & 0 & 0 & 1 & & & & \\ \hline 0 & 0 & 0 & 0 & 1 & 0 & T & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & T \\ k & 0 & 0 & 0 & -k & 0 & 1 & 0 \\ 0 & k & 0 & 0 & 0 & -k & 0 & 1 \end{array}\right] \tag{3-54}$$

where k is a constant.

We could use an approach similar to this example to model the merging of aircraft to form a formation of the type described in Example 1. In this case we would have discrete states M (merge) and $\overline{M}$ (no merge), and we would have transitions as shown in Fig. 3-3. Note that $\overline{M}$ plays the role of $\overline{I}$, I, $\overline{K}$, and M plays the role of K. In this case, however, the model corresponding to M would be the same as in Example 1. Note that the probability of the transition $\overline{M}$ to M depends on the continuous state in the same way that $P(I|\overline{I},x)$ does. This dependence of discrete state transitions on continuous states makes the hybrid state estimation problem more complicated. The optimal Bayesian algorithm no longer has the simple form it has when there are no such dependences. Nevertheless, straightforward suboptimal solutions do exist.

3.3 MULTIOBJECT TRACKING ALGORITHMS FOR FORMATION TRACKING

In this subsection we will discuss multiobject tracking algorithms for the formation tracking problems formulated in the previous section. The basic structure of the algorithm, as described earlier in Section 2, remains the

same, and therefore, we will limit our discussion to those features of the algorithm which are unique to formation tracking.

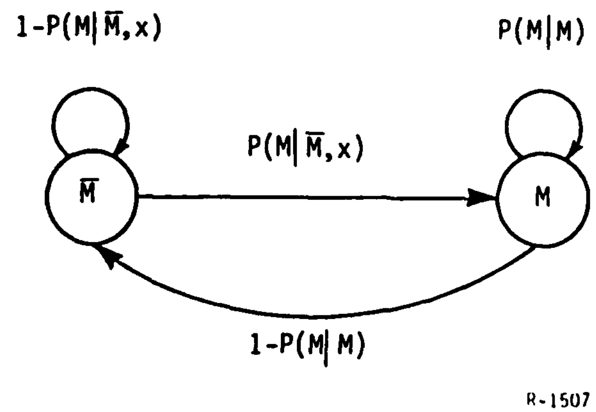

Figure 3-3. Discrete State Transitions for Merging Aircraft.

3.3.1 Optimal Algorithm Structure

The general form of the optimal Bayesian estimation algorithm for the linear Gaussian hybrid state estimation problem (in which discrete state transitions are independent of continuous states) is well known [1],[10]-[14] and was described earlier in Section 2. This algorithm computes the sufficient statistics needed to describe the conditional probability distribution of discrete and continuous states at specified times, given measurements up to that time. The sufficient statistics consist of the probability distribution of the different discrete hypotheses (i.e., the sequences of discrete states), and the conditional means and covariances of the continuous states based on

individual discrete hypotheses. The continuous state statistics are computed using conventional recursive linear estimators (i.e., Kalman filters). What is always difficult in hybrid state estimation problems is organizing the increasing number of discrete hypotheses in an efficient manner. We accomplished this in our earlier work [12], [14] using the track-oriented data structure described earlier in Section 2. In this subsection we describe the optimal multiple formation tracking algorithm in terms of the same track-oriented data structure. We will describe only the structure of the algorithm and not the details of computation in this subsection since the computations (i.e., likelihood and filter computations) are similar to those in our earlier work [12]. The general approach was also reviewed in Section 2.

Formations complicate the multiobject tracking problem in two ways: (1) multiple returns per scan per formation, and (2) correlated dynamics within a formation. We first discuss the effect of 1 on the optimal tracking algorithm structure. Note that the multiobject tracking problems presented in our previous work [12], and in Section 2 do contain one formation, namely the formation consisting of all the false returns or clutter (see Example 1 in the previous subsection). Each <u>target</u> track (or local hypothesis)* associates 1 0 or measurements per scan. Each clutter <u>formation</u> track (or local hypothesis) associates 0,1,2,..., or m(t) measurements per scan where m(t) is the total number of measurements in scan period t. Each global hypothesis contains a complete, consistent set of local hypotheses. It is consistent in that no two local hypotheses contain the same measurement; and it is complete in that the total number of measurements in the set of local hypotheses is m(t).

*The concept of track and local hypothesis is explained in Appendix C.

The situation is essentially the same for multiple formations. At the top of the data structure hierarchy is the list of global hypotheses. Each global hypothesis is a complete set of consistent local hypotheses. Each local hypothesis corresponds to an individual formation and determines a sequence of discrete states relevant to a specific formation. In particular, the local hypothesis specifies the measurements associated with the corresponding formation.

If a formation is more complicated than a clutter formation (and thus contains two or more targets) then the local hypothesis must also indicate how the multiple measurements are associated with targets within the formation. One can represent these different hypotheses efficiently in our track-oriented data structure by extending the track-oriented structure from a two-level to a three-level hierarchy. The hierarchy (shown in Fig. 3-4) consists of global hypotheses at the highest level, local formation hypotheses at the intermediate level, and local target hypotheses at the lowest level. Note that we must include clutter formations (such as Example 1) at the lowest level of the hierarchy together with targets. In this structure each global hypothesis is in fact a set of pointers that link together a consistent set of formation hypotheses. Each formation hypothesis is also in fact a set of pointers that link together a consistent set of target and clutter hypotheses. It is only at the target level that the hypotheses describe the association of measurements with targets or clutter formations.

We have now described how the problem of multiple returns per formation per scan can be incorporated into the track-oriented data structure. Let us now discuss the impact of correlated motion within the formation on the structure of the tracking algorithm. Recall that in the multiobject tracking

algorithm without formations (see Section 2), the sufficient statistical information concerning discrete and continuous states can be stored at the target local hypothesis level. This is no longer true in the formation tracking problem. In the formation tracking problem some discrete states (such as those indicating when targets separate or merge) are associated only with a formation and not with any individual target within a formation. The statistics (i.e., likelihoods) of these states must be stored at the formation local hypothesis level, but this presents no difficulty. Complications arise in formation track ing because the statistics of target continuous and discrete states depend on formation hypotheses as well as the target hypotheses. In other words, the statistics of a specific target's continuous states and discrete states cannot be computed from knowing only the local target hypothesis (such as the association of measurements with the target). One also needs to know the local formation hypothesis. Consequently, in formation tracking problems statistical information concerning target states must be stored and updated at the formation local hypothesis level. This complicates the algorithm, but it is unavoidable in the optimal algorithm.

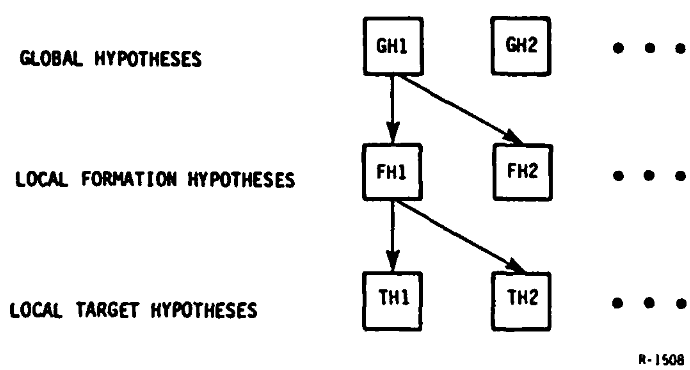

Figure 3-4. Three-Level Track-Oriented Hierarchy.

To better understand the problems involved, consider Example 2 in which an aircraft launches a missile. For simplicity we will assume that targets can appear randomly but do not disappear, that there are no false alarms or missed detections, and that all aircraft carry missiles. Suppose that one measurement (M1) was received at scan t=1 and two measurements (M2 and M3) were received at t=2. Figure 3-5 shows the targets and formations, the corresponding local hypotheses, and the global hypotheses at t=1. Figure 3-5 also explains the notation we use to label the various types of hypotheses. At t=1 there is only one global hypothesis which is that there is one missile-carrying aircraft and the missile is unlaunched. Figure 3-6 shows how the local and global hypotheses change at t=2. Now there are two targets (T1 and T2), two formations (F1 and F2), and four global hypotheses. The formation F1 corresponds to the aircraft-missile formation of t=1. The formation F2 corresponds to a new aircraft-missile formation appearing for the first time at t=2. Thus, at t=2 the global hypotheses GH1[2] and GH2[2] correspond to having two aircraft and no missiles. The global hypotheses GH3[2] and GH4[2] correspond to having one aircraft which has just fired a missile. In particular, note that global hypot eses GH3[2] and GH4[2] say that the new target T2 is a missile, but GH1[2] and GH2[2] say that T2 is an aircraft. Now we can see why the statistics of continuous states, such as the positions of T1 and T2, have to be stored and computed at the formation hypothesis level and not at the target level. In this example, one computes the continuous state estimate of T2 differently depending on whether T2 is postulated to be a new aircraft appearing at random in the surveillance area or a missile just launched by T1.

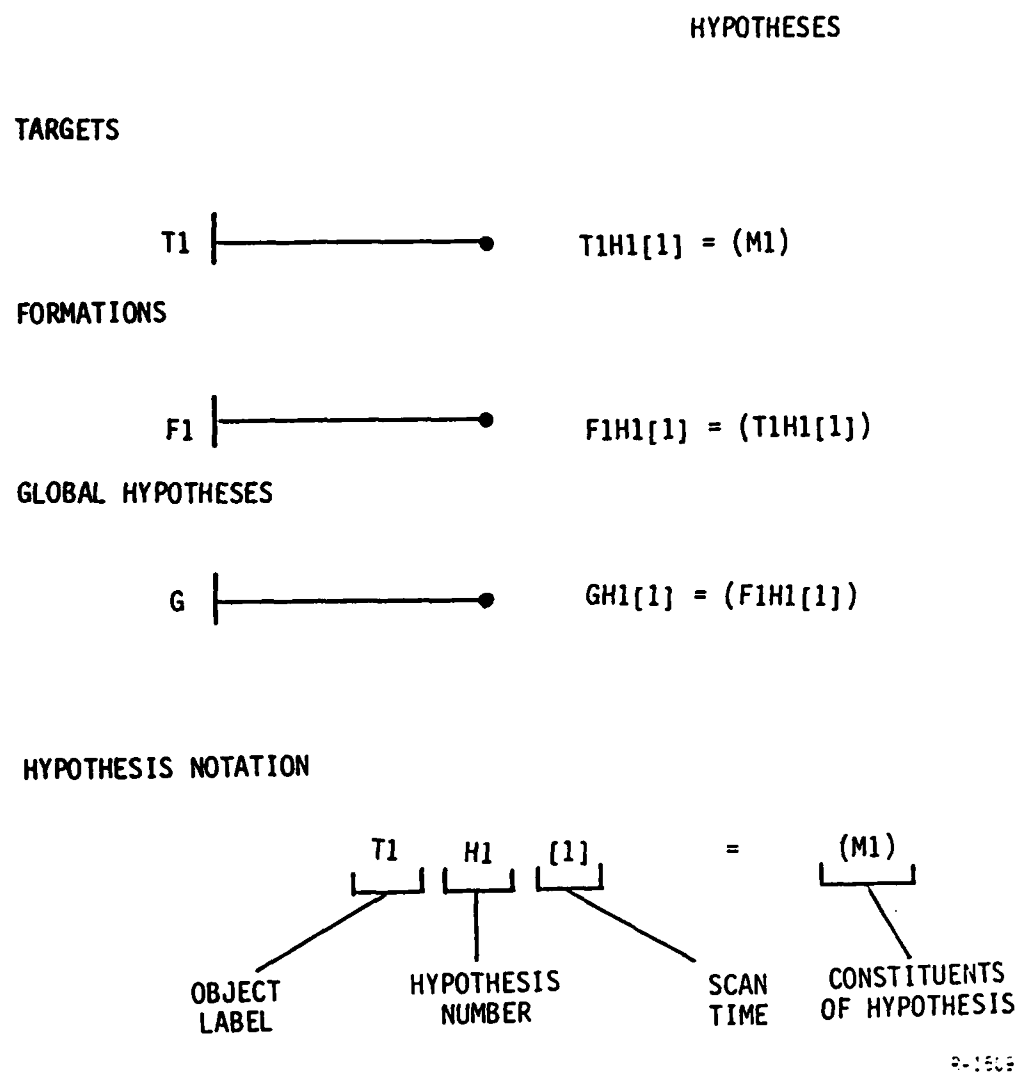

Figure 3-5. Targets, Formations, and Global Hypotheses at t=1.

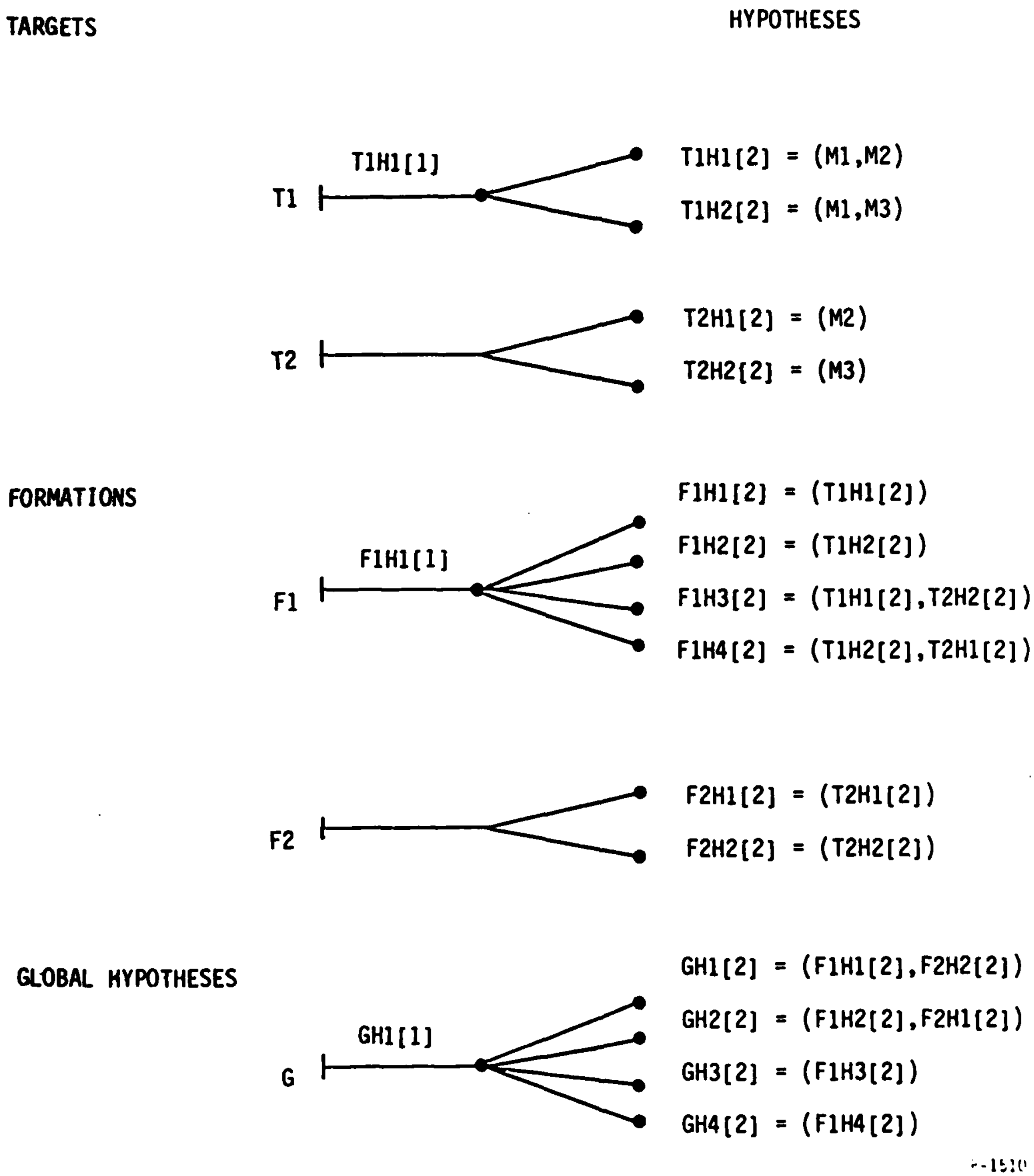

Figure 3-6. Targets, Formations and Global Hypotheses at t=2.

In this subsection we have discussed the structure of the optimal algorithm. This structure has two important features. First, one can use a three-level track-oriented data structure to associate measurements with objects in an efficient manner. Second, one can no longer update the statistics of continuous and discrete states at the target level; it is necessary to do this at the formation level. It is this second feature that makes formation tracking problems considerably more complicated than conventional multiobject tracking problems. In the next subsection we will see that a similar complication arises when we try to apply suboptimal hypothesis management techniques such as screening, pruning, or merging.

3.3.2 Suboptimal Algorithm Structure

In this subsection we will study the structure of suboptimal multiobject tracking algorithms for formation tracking problems. As before we will discuss this structure in terms of the three-level track-oriented data structure described in the last subsection.

Suboptimal multiobject tracking algorithms trade performance for computational feasibility by applying various techniques to reduce the number of hypotheses one creates and stores. The principal methods of hypothesis reduction are screening and pruning (see Section 2). In conventional multiobject tracking without formations one can apply screening at the target local hypothesis level during the creation of new target track nodes from measurement associations. However, in formation tracking this is not generally possible because the continuous state statistics necessary to define a measurement gate are computed and stored at the formation local hypothesis level and not at the target level. Because a target may belong to more than one

formation, there may be several measurement gates associated with each target track node (i.e., each target local hypothesis). It is necessary to compute the measurement gate parameters at the formation local hypothesis level, but it may be more efficient to store these parameters at the target local hypothesis level where new measurements are being associated with targets. In general, a specific target local hypothesis may have several gates and one associates a new measurement with this hypothesis if the measurement falls within any of these gates. In some cases it may be possible to identify one measurement gate that contains all others and use this to screen new measurements.

In conventional multiobject tracking problems without formations one can apply pruning techniques at the target hypothesis level or at the global hypothesis level after the likelihoods of each local or global hypothesis has been computed. In the case of formation tracking these likelihoods are not computed at the target hypothesis level but at the formation and global hypothesis levels. Thus, in the case of formation tracking pruning takes place at the formation or global hypothesis levels but not at the target hypothesis level. However, note that target local hypotheses can be dropped if all formation hypotheses to which they belong have been pruned. Similarly, formation hypotheses can be dropped if all global hypotheses to which they belong have been pruned.

In this subsection we have discussed the structure of suboptimal algorithms based on screening and pruning. Suboptimal algorithms for formation tracking are similar to those for conventional multiobject tracking with one significant difference. Because continuous and discrete state statistics

(i.e., conditional means, error covariances, and likelihoods) are computed only at the formation hypothesis level, it is necessary to perform suboptimal approximations such as screening and pruning at that level also.

SECTION 4

CONCLUSIONS

4.1 MULTIPLE, MANEUVERING TARGETS

We showed in Section 2 that the hybrid state multiobject tracking algorithm can easily incorporate the tracking of multiple, maneuvering targets. The hybrid state framework was flexible enough to formulate the problem of multiple targets which are "born" (i.e., appear) and "die" (i.e., disappear) at unknown times, and which can maneuver between straight and accelerated flight during the interval in which they are "alive." The model also included the possibility of missed detections of a present target, false detections of clutter, and the unknown association of returns in any one scan with targets in track. We also considered the effect of nonlinear radar measurements (i.e., range and azimuth) on the tracking of multiple maneuvering targets.

Although we constructed the optimal hybrid state estimation algorithm for this problem, we found that the additional hypotheses generated to account for the possibility of maneuvers makes it impossible to run the optimal algorithm for more than two or three scans. Thus, we used gating, pruning, and a simple form of merging to reduce the number of hypotheses generated to a manageable number. Two approximations were particularly useful to deal with multiple maneuvering targets. The first approximation was to prevent initiation of maneuvers until the target's velocity error is sufficiently small (as defined

by a prespecified threshold). This screening technique prevented the inclusion of tracks which postulate maneuvers as a consequence of the large gate sizes at the time of track initiation. If a target maneuvers at the time of birth, it is picked up as a new target with little loss of information caused by dropping the previous track. The other useful approximation is to prevent hypothesizing several different maneuvers in succession. Note that both of these techniques for practical implementation can be thought of as resulting from specific hybrid state modeling assumptions. For example, the first results from assumptions about the number of detections one expects before the target can transition to a maneuver state, and the second approximation corresponds to restrictions on the transitions between different maneuver states. This suggests more generally that we can associate particular hybrid state models with many practical algorithms which have been designed using intelligent heuristics and which were not originally based on any well-defined model. This association could clarify the assumptions on which a given practical algorithm is based and thus help to distinguish between different algorithms.

4.2 MULTIPLE FORMATIONS

In Section 3 we further demonstrated the power of the hybrid state approach by generalizing the multiobject tracking problem to include the problem of tracking multiple formations. Multiple formations arise naturally in real surveillance problems and can be used to model such phenomena as aircraft flying in formation, aircraft launching missiles, and missiles intercepting aircraft or other missiles. The problem of tracking formations is generally more difficult than tracking individual targets because several

returns in a single scan may be associated with one formation and because the dynamics of objects within a formation are correlated with each other.

We found that the track-oriented data structure developed in our previous work [12] and clarified in this report can be generalized to deal with multiple formations. This extension resulted in a three-level hierarchical structure consisting of global hypotheses at the highest level, local formation hypothesis at the intermediate level, and individual target and clutter hypotheses at the lowest level. It is only at the lowest level that hypotheses describe the association of measurements.

In the multiobject tracking algorithm without formations, the sufficient statistical information concerning discrete and continuous states could be computed and stored at the target local hypothesis (i.e., target track) level. We found that this is no longer true in the formation tracking algorithm. In the formation tracking problem some discrete states (such as those indicating when targets separate or merge) are associated only with a formation and not with any individual target within a formation. Complications arise because the statistics of target continuous and discrete states depend on formation hypothesis as well as the target hypotheses. Thus, the statistics of a specified target's continuous states and discrete states cannot be computed from knowing only the local target hypothesis (such as the association of measurements with the target). One also needs to know the local formation hypothesis.

These issues make the principal suboptimal methods, gating and pruning, difficult to apply to multiple formation problems. In conventional multi-object tracking one can apply screening at the target local hypothesis level during the creation of new target track nodes from measurement associations.

In formation tracking this is not generally possible because the continuous state statistics necessary to define a measurement gate are computed and stored at the formation local hypothesis level and not at the target level. Because targets may belong to more than one formation, there may be several measurement gates associated with each target track node. Difficulties also arise in pruning because the likelihood statistics used for pruning must be computed and stored at the formation level of the data structure and not at the target track level.

4.3 HYBRID STATE APPROACH

In this report we have sought to illustrate the effectiveness of the hybrid state approach in formulating multiobject tracking problems in particular and airborne surveillance problems in general. The hybrid state formulation provided a useful framework both for modeling many different surveillance problems (e.g., maneuvering targets, formation flying, etc.) and also for constructing theoretically optimal and efficient suboptimal algorithms.

In terms of modeling, the hybrid state approach provides a very general and convenient representation of stochastic dynamic systems. This approach allows us to represent continuous, numerical-valued variables and discrete, logical-valued variables in a systematic, unified framework. We showed the power of this framework to represent both conventional (multiple targets) and novel (multiple formations) surveillance problems. We feel that this ability to treat continuous and discrete variables together in a systematic way makes the hybrid state approach a powerful technique for modeling surveillance problems, and ultimately, for designing surveillance systems.

Once the hybrid state model is specified, optimal and suboptimal estimation algorithms can be derived systematically. Computationally, this algorithm consists of simple likelihood calculations and linear Gaussian estimation computations. All of the complexity of the algorithm lies in the rapidly growing dimensionality of the computations as new discrete hypotheses are generated. The main problem of the algorithm designer is to determine an efficient structure for representing and generating the myriad hypotheses and to find effective techniques for managing the growth of these hypotheses. In this report we have defined the track-oriented data structure which is a very efficient structure for representing the type of hybrid state systems arising in multiobject tracking problems. This structure is convenient both for new hypothesis generation and for likelihood and filter computations. Furthermore, although we have not studied this aspect in depth, the track-oriented data structure provides a natural parallel computational structure for array processing.

Suboptimal techniques for managing hypotheses in the general hybrid state estimation algorithm are straightforward extensions of gating, pruning, merging, and clustering. These fit naturally into the track-oriented structure. However, the quantitative value of these different suboptimal techniques is still unclear. We have performed numerical simulations which give some qualitative picture of the effectiveness of a technique, but much remains to be done in determining quantitative effectiveness.

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APPENDIX A

SOME MATHEMATICAL MODELS FOR MANEUVERS

A nonmaneuvering target is modeled simply in terms of constant velocity motion with small, white-noise accelerations. The state $\underline{x}$ consists of two position coordinates, p_x and p_y, and two velocity coordinates, v_x and v_y. Thus,

$$\underline{x} = \begin{bmatrix} p_x \\ p_y \\ v_x \\ v_y \end{bmatrix} . \tag{A-1}$$

The state transition matrix from scan number k to k+1 is expressed as follows in terms of the time per scan, T

$$\Phi(k+1,k) = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \tag{A-2}$$

The Gaussian white noise $\underline{\xi}_1$ accounts for small perturbations in target velocity. We assume that the sequence of $\underline{\xi}_1(k)$ is Gaussian, uncorrelated, zero-mean, and has a diagonal covariance matrix given by

$$E\{\underline{\xi}_1(k)\underline{\xi}_1(k)^T\} = \mathrm{diag}[0 \quad 0 \quad q_1 \quad q_2] \quad . \tag{A-3}$$

The resulting state equation for a nonmaneuvering target is

$$\underline{x}(k+1) = \Phi(k+1,k)\underline{x}(k) + \underline{\xi}_1(k) \quad . \tag{A-4}$$

There are two basic types of models of maneuvering targets: those which have only <u>well-defined maneuvers</u> and those which have only <u>random maneuvers</u>. One may consider more general maneuver models as combinations of these two basic types. Let θ denote the onset time of the maneuver. The parameter θ may be modeled as a random variable or as an unknown parameter. Define the function $s(k,\theta)$ as follows

$$s(k,\theta) = \begin{matrix} 1 & k \geq \theta \\ 0 & k < \theta \end{matrix} \quad . \tag{A-5}$$

The <u>well-defined maneuver</u> is modeled in terms of an acceleration vector

$$\underline{u}(k,\theta) = \begin{bmatrix} a_x & s(k,\theta) \\ a_y & \end{bmatrix} \tag{A-6}$$

and a control matrix

$$B = \begin{bmatrix} T^2/2 & 0 \\ 0 & T^2/2 \\ T & 0 \\ 0 & T \end{bmatrix} \quad . \tag{A-7}$$

In Eq. A-6 the acceleration components a_x and a_y may be able to assume several possible values. The resulting maneuver model is given by

$$\underline{x}(k+1) = \Phi(k+1,k)\underline{x}(k) + B\underline{u}(k,\theta) + \underline{\xi}_1(k) \quad . \tag{A-8}$$

The random maneuver model is expressed in terms of a second zero-mean white-noise $\underline{\xi}_2$ with diagonal covariance

$$E\{\underline{\xi}_2(k)\underline{\xi}_2(k)^T\} = \mathrm{diag}[0 \quad 0 \quad q_2 \quad q_2] \tag{A-9}$$

where $q_2 > q_1$. The resulting model is given by

$$\underline{x}(k+1) = \Phi(k+1,k)\underline{x}(k) + \underline{\xi}_1(k) + [\underline{\xi}_2(k) - \xi_1(k)]s(k,\theta) \quad . \tag{A-10}$$

Figures A-1 through A-3 illustrate the velocity component of target motion for each of the three types of maneuvers. Note that well-defined maneuvers model maneuvers due to normal course changes, while random maneuvers may model rapid velocity changes such as occur in jinking.

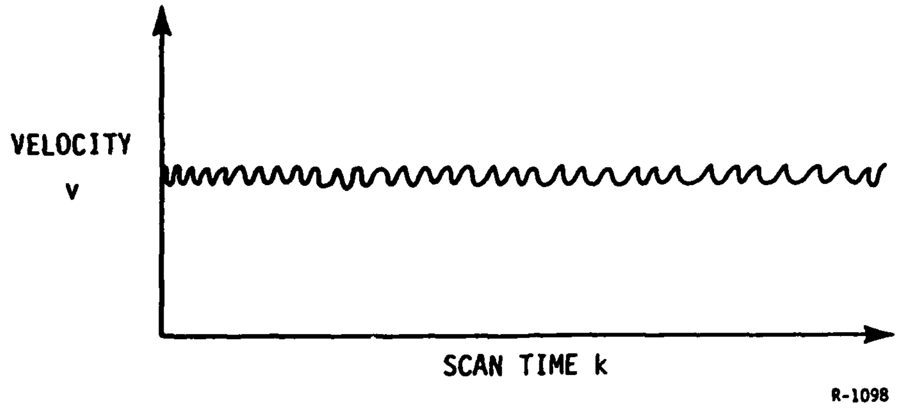


Figure A-1. Nonmaneuvering Target Motion.

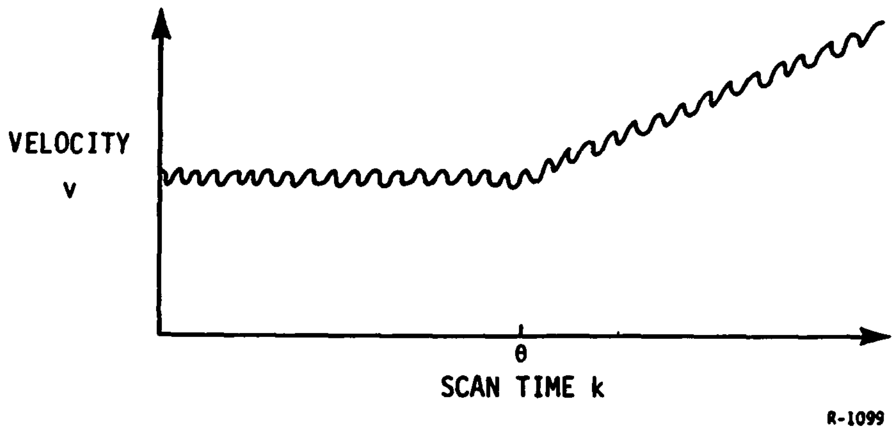


Figure A-2. Well-Defined Maneuver Target Motion.

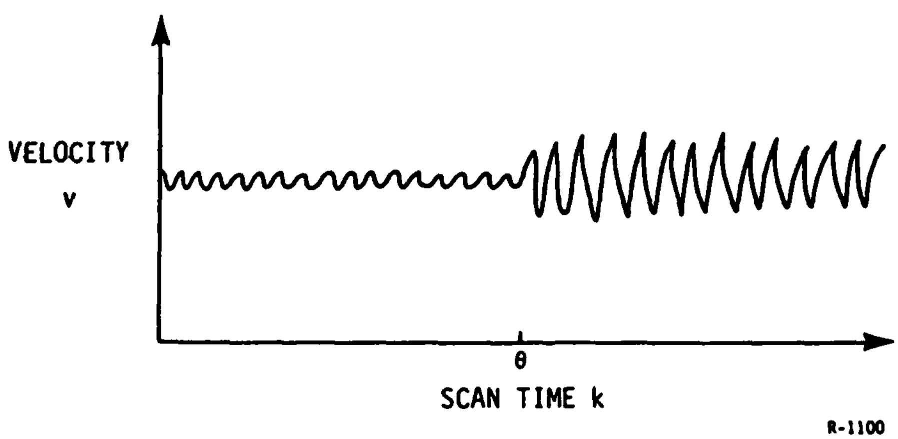


Figure A-3. Random Maneuver Target Motion.

APPENDIX B

SURVEY OF MANEUVER DETECTION AND ESTIMATION ALGORITHMS FOR A SINGLE TARGET

In this appendix we will describe briefly four types of suboptimal approaches to maneuver detection and estimation for single targets. The four approaches are: (1) multiple model adaptive estimation, (2) generalized likelihood ratio, (3) variable state dimension, and (4) minimal-time detection.

B.1 MULTIPLE MODEL ADAPTIVE ESTIMATION

The multiple model adaptive estimation (MMAE) was originally due to Magill [15] where further references are given. This method provides an optimal solution for the hybrid-state estimation problem in which the discrete states do not change with time. In terms of the problem formulation this means that $q(k)=q(1)$ for all k. Such an assumption does not appear to be realistic in maneuver models, but it is still possible to apply the MMAE method by making other realistic, simplifying assumptions. This is the approach taken by [16].

Two assumptions are required to apply the MMAE method to maneuver detection and estimation problems [16]: (1) the probability of a maneuver in any one scan is very small, and (2) the expected time spent in one discrete state before maneuvering to another one is longer than the dynamic response time of the Kalman filter for the continuous states. These assumptions are realistic

for many problems and lead to a feasible implementation. The algorithm consists of the following elements (refer to [16]): (1) a bank of parallel linear filters, one for each discrete state (i.e., type of maneuver); (2) a linear equation to update the posterior discrete state probabilities (the order of this equation is the number of discrete states); and (3) the state estimate is the weighted sum of the filter estimates from 1 using weights from 2. Note that this method provides a single, weighted average state estimate and does not detect the onset time of a maneuver.

B.2 GENERALIZED LIKELIHOOD RATIO

The generalized likelihood ratio (GLR) method was developed by Willsky and Jones [17] to do detection and estimation in systems subject to abrupt changes. A maneuvering target is an example of such a system. This method provides a suboptimal solution of the hybrid state estimation problem in which the discrete states change only once at an unknown or random maneuver time. In terms of the problem formulation this means that

$$q(k) = \begin{matrix} q_1 & , & k<\theta \\ q_2 & , & k>\theta \end{matrix} \quad . \tag{B-1}$$

In Eq. B-1 the maneuver onset time is denoted by θ. The type of maneuver, which is determined by the discrete states q_1 and q_2, is a random state. Note that GLR also permits one to treat continuous random parameters in the maneuver model (e.g., unknown maneuver size).

The assumption of only one maneuver, at an unknown time, is realistic when maneuvers are infrequent. However, even given this assumption, the optimal maneuver detection and estimation algorithm is computationally

infeasible. The GLR method is a suboptimal algorithm that operates as follows [17]: (1) a maneuver is detected, and the maneuver onset time and type are estimated using a likelihood ratio method which does not require knowing the magnitude of the maneuver; (2) the maneuver size is estimated optimally given the onset time and maneuver type; and (3) the tracking algorithm is compensated using maneuver time, type, and size. Note that Step 1 is a reasonable but suboptimal approach to detecting a maneuver and estimating the maneuver onset time.

B.3 VARIABLE STATE DIMENSION

By adding acceleration components to the target state it is possible to design a filter that will track maneuvering targets [18]. However, this state-augmented filter tracks more poorly than the original reduced state filter when the target is not maneuvering and has constant velocity. Bar-Shalom and Birmiwal [19] developed a variable state dimension method to deal with this problem.

The variable state dimension method does not rely on statistical models of the maneuver process. In terms of the problem formulation this means that the discrete state $q(k)$ is a deterministic, but unknown, parameter. Moreover, $q(k)$ takes only two values (maneuver or nonmaneuver) and the corresponding state equation has two different dimensions (6 or 4).

The variable state dimension approach is not based on an optimal criterion (i.e., it is not a Bayesian approach) but employs a reasonable, if ad hoc, statistical approach to detect and estimate maneuvers. The algorithm works as follows [19]: (1) use a chi-square test on the innovations of the four-state constant-velocity filter to detect maneuvers; (2) if a maneuver is

detected, switch to the six-state acceleration filter; (3) if the estimated accelerations are not significant compared to their standard deviation, return to the nonmaneuver state. Note that the computational requirements of this approach are very low. A Monte-Carlo statistical analysis of this method is presented in [19].

B.4 MINIMAL-TIME DETECTION

Under special simplifying assumptions concerning the maneuver process, it is possible to determine an algorithm to detect a maneuver with minimum expected delay for a given false alarm rate. This method is based on the classical quickest detection problem of statistics and is developed by Balikrishnan [20].

The minimal-time detection approach assumes that there are only two discrete states, maneuver and nonmaneuver, and the transition from nonmaneuver to maneuver occurs at a random time. In terms of the problem formulation the maneuver process is similar to that used by the GLR method except that there are only two discrete states. That is,

$$q(k) = \begin{cases} \text{nonmaneuver} & , \ k<\theta \\ \text{maneuver} & , \ k>\theta \end{cases} . \tag{B-2}$$

Let τ denote the time at which the maneuver is detected. Define a function $\chi(s)$ as

$$\chi(s) = \begin{cases} 0 & s<0 \\ s & s>0 \end{cases} . \tag{B-3}$$

The objective of the minimal-time detection method is to minimize the expected time delay,

$$E\{\chi(\tau-\theta)\} \tag{B-4}$$

for a given false alarm probability. Note that a false alarm occurs when $\tau<\theta$.

The optimal detection strategy compares the conditional probability of a maneuver,

$$P\{\theta<k|\underline{z}^k\} \tag{B-5}$$

to a fixed threshold, depending on the desired false alarm rate. One declares a maneuver has occurred when the probability in Eq. B-5 exceeds this threshold.

In order to obtain an implementable solution, one makes a further simplifying assumption concerning the maneuver onset time θ. That is, one assumes that it is geometrically distributed as follows

$$P\{\theta=k\} = \begin{cases} p_0 & , \quad k=1 \\ p(1-p)^{k-1}(1-p_0) & , \quad k>1 \end{cases} . \tag{B-6}$$

Even with this assumption the computational effort involved in computing the conditional probability in Eq. B-5 is considerable.

In principle one may formulate a minimal-time detection problem for more complex maneuver processes. For example, one could allow more than one type of maneuver as in the GLR approach. However, in such a case the optimal detection strategy is not generally a threshold test as it was in the case of one kind of maneuver. At the present time such optimal strategies are computationally infeasible and little is known about what are good suboptimal strategies.

APPENDIX C

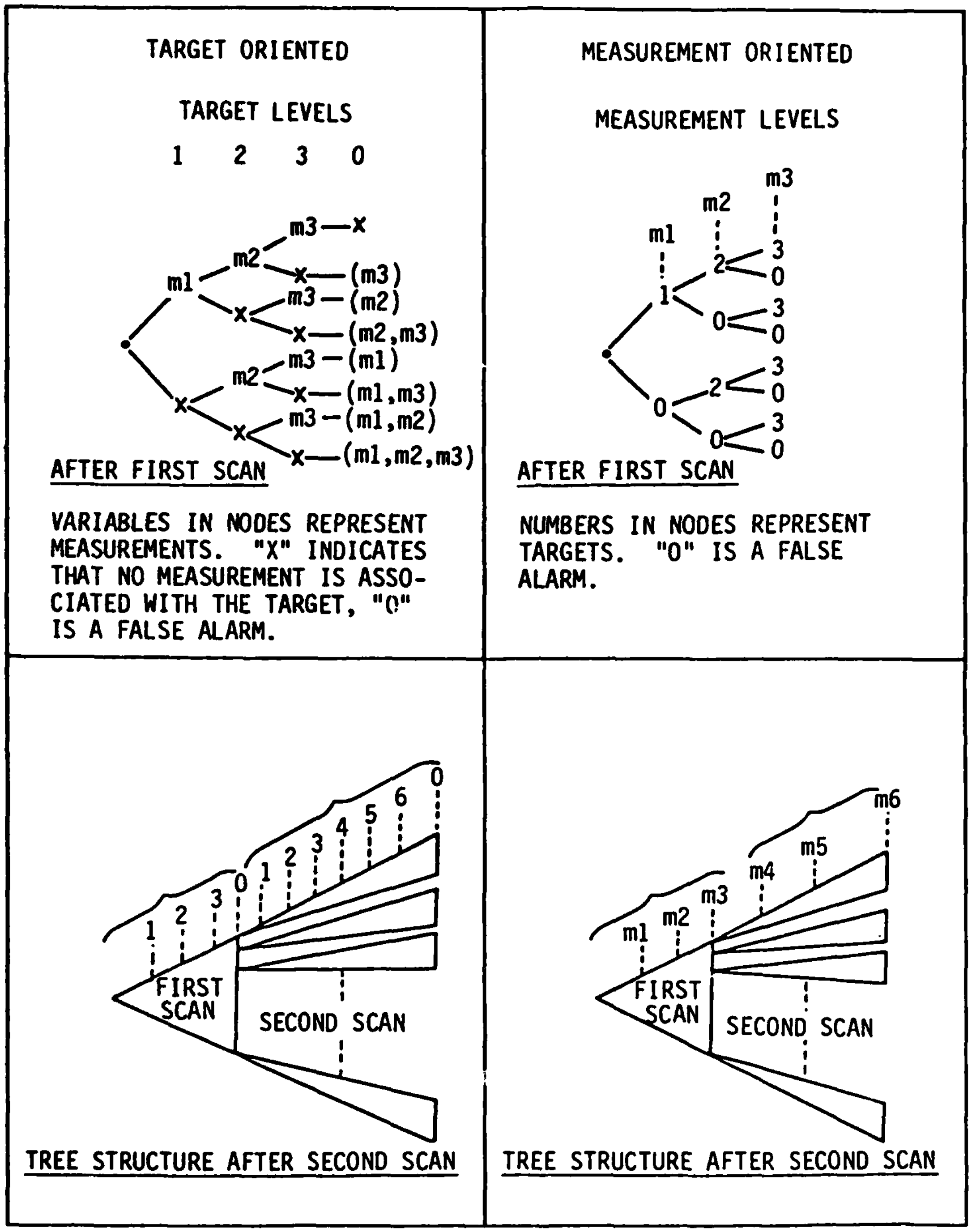
DATA STRUCTURES IN MULTIOBJECT TRACKING ALGORITHMS

This appendix is to clarify the distinction between target- and measurement-oriented data structures which are used in multiobject tracking. Efficient implementation of either approach suggests a third type of data structure which we call track-oriented.

C.1 DEFINITION OF BASIC DATA STRUCTURES

Both target- and measurement-oriented data structures serve to associate measurements and targets in a single hypothesis tree. Figure C-1, taken from Keverian and Sandell [2], illustrates the tree structures corresponding to these two approaches. In each structure a branch of the tree corresponds to a single hypothesis. However, the levels of the tree correspond to targets in the target-oriented data structure and to measurements in the measurement-oriented approach. Correspondingly, the nodes in the tree correspond to measurements in the target-oriented approach and to targets in the measurement-oriented approach. Table C-1 summarizes these differences.

We can represent the two data structures described above conveniently in terms of arrays which express the functional relationship between targets, measurements, and hypotheses. One can view a tree as a relationship between branches, levels, and nodes. For a given branch and level there corresponds a unique node in a tree. Thus, we can represent the tree as a function whose



(a) Target-Oriented Hypotheses (b) Measurement-Oriented Hypotheses

Figure C-1. Target- and Measurement-Oriented Data Structures.

independent variables are branch label and level label and whose dependent variable is node label. Let us write this in the following way:

$$\text{NODE} = \text{NODE(BRANCH,LEVEL)} \quad . \qquad \text{(C-1)}$$

The functional description is equivalent to representing the tree by an array whose elements are node labels and whose indices are branch and level labels.

TABLE C-1. TARGET- AND MEASUREMENT-ORIENTED TREE STRUCTURE COMPARISON

Branches	Hypothesis	Hypothesis
Levels	Target	Measurement
Nodes	Measurement	Target

In terms of the tree representation described above, target-oriented data structures correspond to arrays

$$\text{MEAS(HYP,SCAN,TARG)} \qquad \text{(C-2)}$$

and measurement-oriented data structures correspond to arrays

$$\text{TARG(HYP,SCAN,MEAS)} \quad . \qquad \text{(C-3)}$$

In Eqs. C-2 and C-3 the variable HYP denotes hypothesis label, SCAN denotes scan label (i.e., scan number), TARG denotes target label, and MEAS denotes measurement label. Note that the pair SCAN, TARG in Eq. C-2 are needed to define a unique level label in the target-oriented tree. Similarly, if MEAS is the label of a return within a scan, the pair SCAN, MEAS are needed to define a unique level label in the measurement-oriented tree.

C.2 TARGET-ORIENTED DATA STRUCTURE

The target-oriented data structure corresponds to the array of measurement labels shown in Eq. C-2. What hypotheses can the target-oriented approach describe conveniently? Let MEAS=0 be the label for a no-information measurement (i.e., missed detection). In the target-oriented approach, hypotheses correspond to measurements associated with different targets. The representation implicitly assumes that only one measurement corresponds to a target in a single scan for a given hypothesis. For this reason it is difficult to represent false alarms. To do so requires adding many target labels corresponding to false alarms (no-target target labels!). Let TARG=01, 02, etc. be these false alarm labels. We need several of them in order to preserve the functional form of Eq. C-2, i.e., so that only one measurement label corresponds to a target label in a single scan for a given hypothesis. Having to introduce many false alarm labels is a disadvantage of the target-oriented data structure because one ordinarily cares only whether a measurement is a false alarm or not, but not whether a measurement is a particular false alarm.

Hypothesizing target births is not difficult by itself in the target-oriented approach. To represent such births one needs only to permit a potentially infinite number of target labels, realizing that one will have to store an array whose dimension increases with time but is never infinite dimensional. Target-oriented approaches have difficulty initiating targets because practical problems have target births <u>and</u> false alarms, and because it is necessary to distinguish one from the other. As noted above, the target-oriented data structure can represent both births and false alarms, but false alarms are handled inefficiently and thus realistic target initiation is difficult.

C.3 MEASUREMENT-ORIENTED DATA STRUCTURES

The measurement-oriented data structure is the dual of the target-oriented structure, obtained by interchanging the role of target and measurement labels. Thus, the remarks above for target-oriented structures also hold, interchanging target and measurement, for the measurement-oriented data structure. In particular, let TARG=0 correspond to a nonexisting target (i.e., false alarm). In the measurement-oriented approach, hypotheses correspond to targets associated with different measurements. This representation implicitly assumes that only one target corresponds to a measurement in a single scan for a given hypothesis. For this reason it is difficult to represent missed detections. To do so requires adding measurement labels corresponding to missed detections (no-information measurements). Let MEAS=01, 02, etc. be such labels. We need several of these labels to preserve the functional form of Eq. C-3. This is a disadvantage of the measurement-oriented data structure because one cares only whether a target's detection was missed or not, but not whether the no-detection was a particular miss.

Hypothesizing target deaths is not difficult by itself in the measurement-oriented approach. To represent such deaths one needs a potentially infinite number of measurement labels of the form OM to represent missed detections for a given target which may have died. Note that one will never have to store an infinite dimensional array. Measurement-oriented approaches have difficulty terminating targets because practical problems have target deaths <u>and</u> missed detections, and because it is necessary to distinguish one from the other. Although the measurement-oriented approach can represent both deaths and missed detections, it handles missed detections inefficiently and thus realistic target termination is difficult.

C.4 TRACK-ORIENTED DATA STRUCTURES

In the previous subsections we saw that the target- and measurement-oriented data structures are dually related to each other, and each approach has its corresponding advantages and disadvantages in terms of the efficiency with which the approach can represent certain types of hypotheses. In this subsection we will see that both data structures suggest a third data structure which is more efficient and combines the advantages of the target and measurement-oriented approaches. To begin, consider the target-oriented data structure represented by the array in Eq. C-2. In situations with several targets there will be several different hypotheses which associate the same sequence of measurements with a particular target. For this reason it is possible to represent these hypotheses more efficiently by defining the concept of a local hypotheses for each target. Thus, let us say that two hypotheses, HYP_1 and HYP_2, are equivalent with respect to target label n at scan number t, if

$$MEAS(HYP_1,s,n) = MEAS(HYP_2,s,n) \qquad (C-4)$$

for all s, $1<s<t$. In this case we write

$$HYP_1 \equiv HYP_2(n,t) \quad . \qquad (C-5)$$

Let LOCHYP(n,t) denote the equivalence class of hypotheses under the equivalence relation defined in Eqs. C-4 and C-5. The pair TARG=n, LOCHYP(n,t) define a target track up to scan t. That is, this pair defines a unique sequence of measurements (one return for each scan) with the target label n. The collection of all LOCHYP(n,t) for a given n defines the target track tree

for target label n (see Fig. C-2). For each association of a measurement sequence with target n there corresponds a unique local hypothesis.

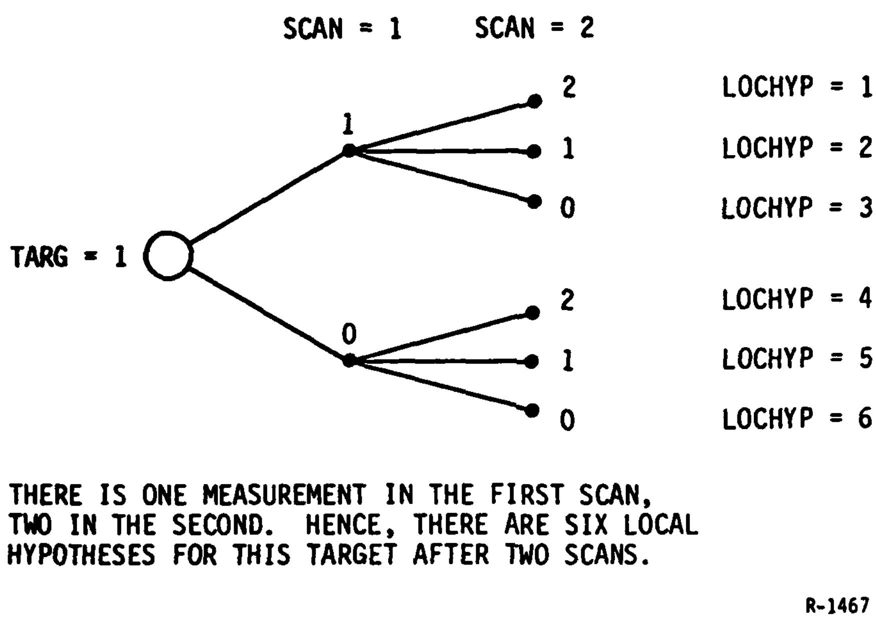


Figure C-2. Target Track Tree for Target 1 After Two Scans.

We can now define a new type of data structure, called a track-oriented data structure, which is equivalent to the target-oriented data structure. Consider a new measurement label array, not to be confused with Eq. C-2, which we denote by

$$\mathrm{MEAS(LOCHYP,SCAN,TARG)} \quad . \qquad \text{(C-5)}$$

Given a target label TARGET=n, Eq. C-5 specifies a tree with branches LOCHYP, levels SCAN, and nodes MEAS. That is, Eq. C-5 represents the collection of different target track trees. In addition to these track trees, one needs the

relationship that associates a pair TARG, LOCHYP with a hypothesis HYP. For example, we can represent this relationship by a collection of pointers which link HYP with associated pairs TARG, LOCHYP. Let us represent the relationship by an array

$$I(HYP,SCAN,TARG,LOCHYP) \tag{C-6}$$

which is 1 if HYP is in the equivalence class LOCHYP(TARG,SCAN) and is 0 otherwise. In other words, I=1 if a target track TARG, LOCHYP belongs to the hypothesis HYP and I=0 otherwise. Note that one need not store Eq. C-6 as an array. Since many entries are going to be 0, one can efficiently store locations of nonzero entries of the array. This is equivalent to defining data pointers (either from hypotheses to tracks, or tracks to hypotheses).

Thus, the track-oriented data structure so far consists of the two arrays in Eqs. C-5 and C-6. Is this new data structure equivalent to the track-oriented structure? Let us consider how false alarms are represented in this new data structure. The target-oriented approach postulates a set of special targets labeled 01, 02, etc. which correspond to false alarms. The construction above of the track-oriented data structure will associate a target track tree with each such label. It should be clear that the different false alarm levels 01, 02, etc. have no real meaning - there are not different types of false alarms. Therefore, define one target track tree corresponding to false alarm (TARG=0). The branches of this tree are hypothesis labels, the levels are scan, measurement pairs, and the nodes are 0 if the measurement if a false alarm for a given scan and hypothesis, otherwise it is 1. This information can be represented by an array

$$I(HYP,SCAN,MEAS) \tag{C-7}$$

which contains only 0 or 1 entries. Note that the array in Eq. C-7 could also be represented by pointers that associate each hypothesis HYP with the collection of measurements (SCAN,MEAS) that are false alarms.

Thus, the track-oriented data structure consists of the arrays in Eqs. C-5, C-6, and C-7. That is, this data structure consists of a collection of target-oriented data structures (the target track trees), a set of links associating hypotheses and tracks, and a measurement-oriented data structure associating measurements with false alarms for each hypothesis. Note that this track-oriented data structure is a combination of target- and measurement-oriented structures. For this reason it can represent both missed detections and false alarms in an efficient manner. The data structure described in Eqs. C-5, C-6, and C-7 contains all information needed to represent all hypotheses at a given scan. Note that in many cases it will be possible to simplify this data structure even further (e.g., one may only need to know the number of false alarms in a scan without having to know the measurements associated with false alarms; in this case the array in Eq. C-7 can be simplified).

C.5 RELATIONSHIP OF MEASUREMENT- AND TRACK-ORIENTED DATA STRUCTURES

In subsection C.4 we constructed a track-oriented data structure and at the same time showed its equivalence to a target-oriented data structure. One can carry out a similar construction starting from a measurement-oriented data structure. We begin by defining local hypothesis (i.e., target tracks) in terms of equivalence classes of hypothesis. Let us say that two hypotheses, HYP_1 and HYP_2, are equivalent with respect to target label n and scan number t, if there exist two measurement label sequences $m_k(s)$, $1 \le s \le t$, $k=1,2$, such

that either both $m_1(s)$ and $m_2(s)$ are missed detections (perhaps with different labels) or both are the same measurement label, for each s, $1 \leq s \leq t$, and for each s

$$n = TARG(HYP_1, s, m_1(s)) = TARG(HYP_2, s, m_2(s)) \quad . \qquad \text{(C-8)}$$

Equation C-8 defines the same equivalence relation as in Eq. 6-4, although the presence of missed detections makes the construction more difficult in this case. Now we can define the arrays in Eqs. C-5 and C-6 just as before. We can construct the array in Eq. 6-7 more easily than before because the measurement-oriented approach treats false alarms more efficiently. Define I(HYP,SCAN,MEAS) to be 0 if TARG(HYP,SCAN,MEAS)=0, otherwise let it be 1.

C.6 CONCLUDING REMARKS

This appendix has shown that target- and measurement-oriented data structures are dually related to each other, and each can be used to represent all hypotheses in multiobject tracking. However, each approach handles some aspects of multiobject tracking more efficiently and other aspects less efficiently (e.g., target-oriented approaches can treat target deaths and missed detections more efficiently than target births and false alarms; measurement-oriented approaches are opposite in these regards). Finally, we showed that both data structures are mathematically equivalent (in the sense of representing the same information) to a third data structure which we described as track-oriented. The track-oriented combines the advantages of both target and measurement-oriented data structures in a more efficient representation of hypotheses in multiobject tracking problems.

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